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Response of Well Aquifer Systems to Earth Tides: Problem Revisited

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Two recent works cause us to reexamine Bredehoeft's (1967) analysis of earthtide response of water wells. Narasimhan et al. (1984) raise several questions regarding Bredehoeft's (1967) analysis and suggest that the analysis is internally inconsistent. They argue that one cannot directly estimate the specific storage, which characterizes the drained behavior of a porous medium, from earth tide response, which is an undrained phenomenon. We resolve the questions raised by Narasimhan et al. (1984) and show that Bredehoeft's analysis is internally consistent. In addition, we show that it is possible to determine the specific storage from undrained loading. While Bredehoeft's analysis is somewhat heuristic and neglects grain compressibility, Van der Kamp and Gale (1983) present a more rigorous analysis that is based on Biot's (1941) constitutive relationships and accounts for grain compressibility. However, their results reduce to Bredehoeft's results when grains are assumed incompressible. This suggests that Bredehoeft's analysis has incorporated all the essential features of Biot's relationships except for grain compressibility. Upon reexamining Bredehoeft's analysis we find that this is indeed the case.

INTRODUCTION

A number of authors have attempted to interpret the earth-tide response of water wells: *Melchior* [1960], *Bredehoeft* [1967], *Bodvarsson* [1970], *Robinson and Bell* [1971], *Marine* [1975], *Arditty et al.* [1978], *Van der Kamp and Gale* [1983], and *Narasimhan et al.* [1984]. *Bredehoeft's* [1967] analysis provides a method for determining the specific storage of the aquifer material if Poisson's ratio of the aquifer material is known. This analysis is somewhat heuristic in that it is not based on *Biot's* [1941] constitutive relationships for a fluid-filled porous medium. Two recent works [*Van der Kamp and Gale*, 1983; *Narasimhan et al.*, 1984] cause us to reexamine Bredehoeft's analysis. The result of this reexamination is the subject of this paper.

Narasimhan et al. [1984] raise a number of questions regarding *Bredehoeft's* [1967] analysis and suggest that the analysis is internally inconsistent. They argue that one cannot directly estimate the specific storage from earth tide response. They base this argument on the fact that a confined aquifer responds to earth tide loading in an undrained fashion, while the specific storage quantifies a drained behavior of a porous medium. In an earlier work, *Narasimhan and Kanehiro* [1980] argue the more general case that the specific storage cannot be directly determined from undrained loading experiments. We show below that it is possible to directly determine the specific storage from undrained loading experiments. Thus it is not unreasonable that one can determine the specific storage from earth tide response. We also resolve the questions raised by Narasimhan et al. and show that Bredehoeft's analysis is internally consistent.

Van der Kamp and Gale [1983] apply *Biot's* [1941] constitutive relationships to analyze the earth tide response of water wells. Their analysis accounts for compressibility of the grains of the aquifer material and is therefore less restrictive than *Bredehoeft's* [1967] analysis, which assumes that the grains are incompressible. However, Van der Kamp and Gale showed that their result reduces to Bredehoeft's result when the grains are assumed incompressible. This suggests that Bredehoeft's analysis incorporated all the essential features of Biot's constitutive relationships except for grain compressibility. In other words, Bredehoeft implicitly applied Biot's constitutive relationships to his analysis without explicitly stating so. We show below that this is indeed the case.

The rest of this paper is organized as follows. First, we show by two simple examples that, in principle, one can directly determine the specific storage of a porous medium from its response to undrained loading. Second, we summarize the key features of *Bredehoeft's* [1967] analysis. We note, in particular, that there are two sign errors in Bredehoeft's analysis. However, both are printing errors and are not carried through the analysis, so that the final result is unaffected. Third, we show that the key results of Bredehoeft's analysis can be obtained if one starts with *Biot's* [1941] constitutive relationships. Finally, we provide a discussion that resolves the questions raised by *Narasimhan et al.* [1984] and show that Bredehoeft's analysis is internally consistent.

As much as possible, we follow the notations used by *Van der Kamp and Gale* [1983]. In Table 1 we show the equivalence between the notations used in this paper versus those used by *Bredehoeft* [1967] and *Narasimhan et al.* [1984]. The stress and strain components σ_{ij} and ε_{ij} and the pore pressure p denote incremental values or deviations from an undisturbed state. Positive stress denotes compression, and positive strain denotes contraction. This convention is used by *Van der Kamp and Gale* [1983] but is opposite to that of *Biot* [1941] and *Bredehoeft* [1967]. *Narasimhan et al.* [1984] define positive stress as compressive and positive strain as extensive.

The octahedral stress (or mean normal stress) is defined as

$$\sigma_t = (1/3)(\sigma_{11} + \sigma_{22} + \sigma_{33}) \tag{1}$$

while the volumetric strain (or dilatation) is

$$\varepsilon = (\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}) \tag{2}$$

We assumed that the grains are incompressible and that all pore space is continuous and saturated with water. The porous medium is assumed to behave elastically, with elastic moduli independent of stress.

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TABLE 1. Equivalence Between Notations Used in This Paper and Those Used in *Bredehoeft* [1967] and *Narasimhan et al.* [1984]

This Paper	*Bredehoeft* [1967]	*Narasimhan et al.* [1984]
K'	E_s	E_s
K		$1/c_m$
K_f	E_w	E_w, $1/c_w$
n	n	ϕ
p	dp	δp
S_s	S_s	
S'		S_s
$e_{33,h}$	$-\Delta_h$	$-\Delta_h$
$\varepsilon_{t,d}$	$-\Delta_t$	$-\Delta_t$
$\varepsilon_{t,u}$	$-\Delta$	$-\Delta$
σ_t		$\delta\sigma_m$

DETERMINATION OF SPECIFIC STORAGE FROM UNDRAINED LOADING EXPERIMENTS

To begin with, we point out that two expressions for specific storage have appeared in the literature. Following *Van der Kamp and Gale* [1983], but assuming that the grains are incompressible, we write

$$S_s = \rho g\left(\frac{1}{K'} + \frac{n}{K_f}\right) \tag{3}$$

$$S' = \rho g\left(\frac{1}{K} + \frac{n}{K_f}\right) \tag{4}$$

where

- ρ density of water, M/L^3;
- g acceleration of gravity, L/T^2;
- K' confined modulus of elasticity under drained conditions, M/LT^2;
- K bulk modulus of elasticity under drained conditions, M/LT^2;
- K_f bulk modulus of elasticity of water, M/LT^2;
- n porosity.

Equation (3) is the normal expression for specific storage as used in the groundwater literature and considers only vertical deformation, i.e., $\varepsilon_{11} = \varepsilon_{22} = 0$. K' can be determined by a drained loading experiment in which a sample inside a stiff cell is loaded vertically but no horizontal expansion is allowed (see, for example, *Freeze and Cherry* [1979, pp. 54–56]). We refer to this experimental set up as experiment A. The vertical strain ε_{33} can be determined by dividing the change in height of the sample by the original height of the sample. From its definition, K' can be calculated by

$$K' = (\sigma_{33} - p)/\varepsilon_{33} \tag{5}$$

Because the loading is drained, $p = 0$, so that $K' = \sigma_{33}/\varepsilon_{33}$.

Van der Kamp and Gale [1983] call S' as defined by (4) the "three-dimensional storage coefficient." This expression for specific storage has been used by *Narasimhan and Kanehiro* [1980] and *Narasimhan et al.* [1984]. K can be determined by a drained experiment in which the sample is loaded by the external stresses σ_{11}, σ_{22}, and σ_{33} (usually with $\sigma_{11} = \sigma_{22}$). We refer to this experimental set up as experiment B. In principle, ε can be measured, and K is computed by

$$K = (\sigma_t - p)/\varepsilon \tag{6}$$

Again, $p = 0$ for drained loading, so that $K = \sigma_t/\varepsilon$.

We now show that, in principle, S_s and S' can be determined by undrained loading experiments. For a sample with incompressible grains the volume change of the sample under undrained conditions is equal to the volume change of the water in the sample. In other words,

$$\varepsilon = np/K_f \tag{7}$$

First, consider experiment A, but now modified so that there is no drainage. Becuase $\varepsilon = \varepsilon_{33}$, (5) and (7) can be combined to give

$$\frac{1}{K'} + \frac{n}{K_f} = \frac{\sigma_{33}}{pK'} \tag{8}$$

Using (3), (8) can be written as

$$S_s = \frac{\rho g \sigma_{33}}{pK'} \tag{9}$$

Substituting (5) into (9) yields

$$S_s = \frac{\rho g \sigma_{33} \varepsilon_{33}}{p(\sigma_{33} - p)} \tag{10}$$

Equation (10) indicates that if the vertical stress, vertical strain, and pore pressure are known, then the specific storage S_s can be determined from an undrained experiment. In practice, the determination of S_s under the conditions of experiment A may be difficult for highly compressible earth materials such as clays. For such materials the load may be almost entirely born by the fluid, so that the term $\sigma_{33} - p$ in the denominator of (10) approaches zero. Consequently, even small errors in the measurement of σ_{33} and/or p may cause significant errors in the determination of S_s. For materials other than clays, however, the difference between σ_{33} and p should be large enough to yield accurate estimates of S_s.

Next, consider experiment B, also modified so that there is no drainage. Combining (6) and (7) yields

$$\frac{1}{K} + \frac{n}{K_f} = \frac{\sigma_t}{pK} \tag{11}$$

Using (4), (11) can be written as

$$S' = \frac{\rho g \sigma_t}{pK} \tag{12}$$

Substituting (6) into (12) yields

$$S' = \frac{\rho g \sigma_t \varepsilon}{p(\sigma_t - p)} \tag{13}$$

which is the expression for determining S' from an undrained experiment in which the sample is loaded by the external stress σ_{11}, σ_{22}, and σ_{33}. Again, we note that (13) should be applied to materials with sufficiently low compressibilities so that the term $(\sigma_t - p)$ in the denominator of (13) does not approach zero and thus can be accurately determined.

Narasimhan et al. [1984] argue that because the specific storage characterizes a drained behavior, it cannot be determined directly from the undrained response of an aquifer to earth tides. The two examples above show that the specific storage can, in principle, be determined from undrained response. Also, the calculation of specific storage from measurements of undrained phenomena has been accepted by hydrogeologists since *Jacob*'s [1940] analysis of tidal and barometric efficiency. Thus it is not unreasonable that the specific storage can be determined from earth tide response.

KEY FEATURES OF BREDEHOEFT'S [1967] ANALYSIS

A main assumption in *Bredehoeft*'s [1967] analysis is that the horizontal tidal strains, ε_{11} and ε_{22}, in the aquifer (or the latitudinal and longitudinal strains near the Earth's surface) can be determined by independent means. In most cases, these strains will have to be estimated from solid-earth-tide theory, using the overall elastic properties of a radially stratified earth. By comparing theoretical calculations with field measurements, *Beaumont and Berger* [1975] showed that the horizontal tidal strains can usually be estimated from solid-earth-tide theory to an accuracy of about $\pm 50\%$ of the estimate. The uncertainty is due mainly to the effects of ocean tide loading, departure of the Earth from radial stratification, and distortion of the regional strain field by local topography and geologic structure. Correcting for the effects of ocean tide loading, through the use of ocean tide models, can reduce the uncertainty to about $\pm 25\%$ of the estimate.

Another assumption in Bredehoeft's analysis is that the tidal potential does not impose significant vertical stress on the aquifer. This assumption is reasonable because of the proximity of the aquifer to the Earth's surface, which is free of tidal traction. In this case, the vertical tidal strain can be separated into the following two components: (1) the vertical strain produced by the horizontal tidal strains in the absence of pore pressure, i.e., under drained conditions and (2) the vertical strain produced by the change in pore pressure under constant vertical stress and zero horizontal strains. We denote the former quantity by $\varepsilon_{33,t}$ and the latter by $\varepsilon_{33,h}$.

For constant vertical stress, $\varepsilon_{33,t}$ can be computed by

$$\varepsilon_{33,t} = -[\nu/(1-\nu)](\varepsilon_{11} + \varepsilon_{22}) \tag{14}$$

where ν is Poisson's ratio of the aquifer material under drained conditions. The tidal dilatation under drained conditions is thus

$$\varepsilon_{t,d} = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33,t} = [(1-2\nu)/(1-\nu)](\varepsilon_{11} + \varepsilon_{22}) \tag{15}$$

Note that $-\varepsilon_{t,d}$ is Δ_t of *Bredehoeft* [1967], the minus sign being due to the difference in sign convention of the dilatation term.

The tidal dilatation under undrained conditions can be expressed as the sum of the drained dilatation and the vertical strain produced by the change in pore pressure under constant vertical stress and zero horizontal strains. That is,

$$\varepsilon_{t,u} = \varepsilon_{t,d} + \varepsilon_{33,h} \tag{16}$$

Note that $-\varepsilon_{t,u}$ and $-\varepsilon_{33,h}$ are, respectively, Δ and Δ_h of *Bredehoeft* [1967]. Thus (16) is the same as Bredehoeft's equation (21).

From (5) and under the condition of constant vertical stress, i.e., $\sigma_{33} = 0$, we have

$$\varepsilon_{33,h} = -(p/K') \tag{17}$$

Equation (17) should be the same as Bredehoeft's equation (22). As pointed out by *Narasimhan et al.* [1984, p. 1915], there is a sign error in Bredehoeft's equation (22), which should correctly read: $\Delta_h = dp/E_s$. We note that an unnumbered equation of *Bredehoeft* [1967, p. 3082] also contains a sign error and should correctly read: $\Delta_t = -(n\ dp/E_w + dp/E_s)$. However, both are printing errors and are not carried through in the analysis, so that the final result is unaffected.

Substituting (17) and (15) into (16) yields

$$\varepsilon_{t,u} = \frac{1-2\nu}{1-\nu}(\varepsilon_{11} + \varepsilon_{22}) - \frac{p}{K'} \tag{18}$$

For undrained conditions, (7) applies so that we can also write

$$\varepsilon_{t,u} = np/K_f \tag{19}$$

Again, note that (19) is the same as (3) of *Bredehoeft* [1967]. Setting equal the right-hand sides of (18) and (19) yields after some rearrangements

$$\frac{1}{K'} + \frac{n}{K_f} = \frac{(1-2\nu)(\varepsilon_{11} + \varepsilon_{22})}{(1-\nu)p} \tag{20}$$

Using (3), we have

$$S_s = \frac{\rho g(1-2\nu)(e_{11} + \varepsilon_{22})}{(1-\nu)p} \tag{21}$$

Equation (21) is the result of *Bredehoeft*'s [1967] analysis. This can be shown by substituting (15) into (21). After rearrangement we obtain

$$\frac{p}{\rho g} = \frac{\varepsilon_{t,d}}{S_s} \tag{22}$$

which is *Bredehoeft*'s [1967] equation (25).

Equation (21) or (22) indicates that the uncertainty in the estimate of the S_s is directly proportional to the uncertainty in the estimate of the horizontal tidal strain, assuming that Poisson's ratio is known and pore pressure is accurately measured. We noted above that the uncertainty in the theoretical calculation of horizontal tidal strain is approximately $\pm 50\%$. The same uncertainty should apply to the S_s estimate. Although this uncertainty is not insignificant, the S_s estimate should nevertheless be of considerable value in dealing with hydrogeologic problems.

ANALYSIS OF EARTH TIDE RESPONSE USING BIOT'S [1941] CONSTITUTIVE RELATIONSHIPS

We now show that the key features of *Bredehoeft*'s [1967] analysis, described in the previous section, can be obtained if one starts with *Biot*'s [1941] constitutive relationships, which can be expressed as

$$\varepsilon_{ij} = \frac{1}{K}\left[\frac{1+\nu}{3(1-2\nu)}\sigma_{ij} - \frac{\nu}{1-2\nu}\sigma_t\delta_{ij} - \frac{\alpha}{3}p\delta_{ij}\right] \tag{23}$$

In (23), δ_{ij} is the Kronecker delta, and α is, according to *Biot* [1941, p. 159], a parameter that "measures the ratio of the water volume squeezed out to the volume change of the soil if the latter is compressed while allowing water to escape," i.e., under drained conditions. (Here we can replace the word "soil" by "fluid-filled porous medium.") *Nur and Byerlee* [1971] showed that

$$\alpha = 1 - (K/K_s) \tag{24}$$

where K_s is the bulk modulus of the grains.

As before, we assume that ε_{11} and ε_{22} are known and that $\sigma_{33} = 0$. Setting $i = j = 3$ in (23) then yields

$$\varepsilon_{33} = \frac{1}{K}\left(-\frac{\nu}{1-2\nu}\sigma_t - \frac{\alpha}{3}p\right) \tag{25}$$

The octahedral stress σ_t can be expressed in terms of the dilatation ε and the pressure p by summing the $i = j$ terms of (23). This gives

$$\varepsilon = (1/K)(\sigma_t - \alpha p) \tag{26}$$

or

$$\sigma_t = K\varepsilon + \alpha p \tag{27}$$

If the grains are incompressible, then $1/K_s = 0$, and (24) becomes $\alpha = 1$. For this case, substituting (27) into (25) yields

$$\varepsilon_{33} = -\frac{\nu}{1-2\nu}\varepsilon - \frac{1+\nu}{3K(1-2\nu)}p \qquad (28)$$

Using (2), (28) can be rearranged to give

$$\varepsilon_{33} = -\frac{\nu}{1-\nu}(\varepsilon_{11}+\varepsilon_{22}) - \frac{1+\nu}{3K(1-\nu)}p \qquad (29)$$

Van der Kamp and Gale [1983, equation (20)] noted that

$$K' = \frac{3K(1-\nu)}{(1+\nu)} \qquad (30)$$

Thus (29) becomes

$$\varepsilon_{33} = -\frac{\nu}{1-\nu}(\varepsilon_{11}+\varepsilon_{22}) - \frac{p}{K'} \qquad (31)$$

Note that the first term on the right-hand side of (31) is $\varepsilon_{33,t}$, and the second term is $\varepsilon_{33,h}$. In other words use of Biot's constitutive relationships allows one to compute the vertical tidal strain in one step while Bredehoeft's analysis requires two steps by computing $\varepsilon_{33,t}$ and $\varepsilon_{33,h}$ separately.

Using (31), the undrained tidal dilatation is found to be

$$\varepsilon_{t,u} = \varepsilon_{11}+\varepsilon_{22}+\varepsilon_{33} = \frac{1-2\nu}{1-\nu}(\varepsilon_{11}+\varepsilon_{22}) - \frac{p}{K'} \qquad (32)$$

which is the same as the expression obtained by Bredehoeft's analysis (see equation (18)).

Van der Kamp and Gale [1983] also noted that for undrained conditions, p and σ_t are related by

$$\sigma_t = p/\beta \qquad (33)$$

where β is a parameter that defines the portion of the stress increment born by the fluid and can be expressed as

$$\beta = \left(\frac{1}{K}-\frac{1}{K_s}\right)\left[\frac{1}{K}-\frac{1}{K_s}+n\left(\frac{1}{K_f}-\frac{1}{K_s}\right)\right]^{-1} \qquad (34)$$

Substituting (33) into (26) and identifying ε with $\varepsilon_{t,u}$ yields

$$\varepsilon_{t,u} = \frac{1}{K}\left(\frac{1}{\beta}-\alpha\right)p \qquad (35)$$

which is an expression that relates dilatation to pressure change for undrained loading of a medium with compressible grains. For incompressible grains, however, $1/K_s = 0$ and $\alpha = 1$, and (35) reduces to

$$\varepsilon_{t,u} = (np/K_f) \qquad (36)$$

which is identical to (19). By setting the right-hand sides of (32) and (36) equal to one another, Bredehoeft's result given by (21) or (22) is obtained.

The equivalence between the analysis of *Bredehoeft* [1967] and the one based on Biot's constitutive relationships is now apparent. In both analyses the undrained tidal dilatation is computed by two methods. Setting the two resultant expressions equal to one another yields the expression for specific storage. The first method assumes that ε_{11} and ε_{22} can be calculated independently and ε_{33} is expressed as a function of ε_{11}, ε_{22}, and ν. In Bredehoeft's analysis, ε_{33} is computed by $\varepsilon_{33} = \varepsilon_{33,t} + \varepsilon_{33,h}$, where $\varepsilon_{33,t}$ is given by (14) and $\varepsilon_{33,h}$ is given by (17). In the analysis based on Biot's relationships, ε_{33} is computed directly from (25). However, the two analyses yield identical results. The second method for computing $\varepsilon_{t,u}$ is to relate $\varepsilon_{t,u}$ to pressure change p in the aquifer. Bredehoeft's analysis uses (19). In the analysis based on Biot's relationships, (35) holds for a medium with compressible grains. However, for incompressible grains, (35) reduces to (19), and thus the two analyses again yield identical results.

DISCUSSION

Narasimhan et al. [1984] suggest that Bredehoeft's analysis is internally inconsistent. In particular, they raise three issues that question the validity of Bredehoeft's result. The issues are: (1) equation (25) of Bredehoeft (or equation (22) of this paper) contains a contradiction, (2) a sign error in equation (22) of Bredehoeft leads to a result that is different than Bredehoeft's result, and (3) Bredehoeft's analysis requires that the aquifer respond simultaneously in a drained and undrained fashion: This requirement is physically implausible. With regard to the second issue we pointed out above that the sign error in Bredehoeft's equation (22) is not carried through the analysis and so the final result is not affected. In the following discussion we resolve the remaining two issues raised by *Narasimhan et al.* [1984] and show that Bredehoeft's analysis is internally consistent.

In their discussion of Bredehoeft's equation (25), or (22) in this paper, *Narasimhan et al.* [1984, p. 1915] write: "Bredehoeft assumes that the tidal strains that define Δ_t are independent of the elastic properties of the aquifer and are almost entirely determined by the elastic properties of the earth as a whole [*Bredehoeft*, 1967, p. 3081]. Yet in his equation (25) ..., Δ_t is related to S_s, which is clearly a function of the aquifer parameters. There is a contradiction here." This, however, is a misleading account of Bredehoeft's analysis. The fact that the tidal dilatation and the specific storage appear in the same equation does not imply that they are "related" to one another, in the sense that one can be determined from the other. In fact, the quantity that is related to the specific storage is the hydraulic head change (dh in Bredehoeft's equation (25) or $p/\rho g$ in equation (22) of this paper). Bredehoeft's equation (25) can be interpreted as follows: Given that the tidal dilatation can be estimated a priori, the magnitude of the hydraulic head fluctuation due to tidal effects is inversely proportional to the specific storage of the aquifer material. Recall from (3) that the specific storage is a function of K', K_f, and n. If K' and K_f are assumed constant, then the magnitude of the hydraulic head fluctuation will be inversely proportional to the porosity n of the aquifer material. *Bredehoeft* [1967, pp. 3083–3084], for example, noted that since deep aquifers tend to have lower porosities than shallow aquifers, it would be reasonable to expect that deep wells should exhibit larger tidal fluctuations than shallow wells completed in the same rock type.

The third issue raised by *Narasimhan et al.* [1984, p. 1915] is stated as follows: "The basic implication in Bredehoeft's development is that one can simultaneously measure (1) '... the dilatation that would occur if the fluid were not present' [*Bredehoeft*, 1967, p. 3080] and (2) the pore pressure generated due to earth tides. This is not possible because the two events are mutually exclusive." However, the need to simultaneously measure the above two quantities is not implied by Bredehoeft's analysis. The reasoning behind Bredehoeft's analysis is that the undrained tidal dilatation can be considered to be produced by the sum of two processes: (1) tidal loading under drained conditions and (2) increase in pore pressure under constant total stress and zero horizontal strain. This line of reasoning does not require that the two processes operate simultaneously in reality. An analogy in well hydraulics serves to clarify the point. In analyzing the recovery of a well after a period of constant pumping one can consider the hydraulic head in the well to be produced by the sum of two actions: (1) the well continues to be pumped at the constant rate prior to shut-in and (2) water is injected into the well at the same rate as pumping. However, it is not necessary to require that, in reality, both pumping and injection take place at the same time.

Alternatively, the undrained tidal dilatation can be viewed as a two-stage experiment. This is similar in concept to the two-stage experiment described by *Narasimhan et al.* [1984, p. 1914], but we shall consider strain rather than stress as the controllable quantity (or independent variable) in this experiment. During the first stage, the sample is subjected to sufficiently large compressive horizontal stresses to cause (positive) horizontal strains of ε_{11} and ε_{22}. The vertical stress is unchanged, and the vertical strain is $\varepsilon_{33,r}$, which is negative because the sample elongates in this direction. Water is allowed to drain out of the sample. During the second stage, the horizontal strains are maintained, and the drained fluid is forced back into the sample. The pore pressure increases, and the vertical strain changes additionally by $\varepsilon_{33,h}$ (also negative, so the sample further elongates). If the assumptions of linear elasticity hold, then the final state of the sample after this two-stage experiment is the same as the final state obtained by executing the first stage under undrained conditions. Bredehoeft's equation (21) (equation (16) in this paper) can be viewed as a mathematical expression of this idea.

In closing we wish to emphasize that our discussion is not aimed at criticizing the tidal analysis of *Narasimhan et al.* [1984] but to resolve the issues raised by these authors regarding *Bredehoeft*'s [1967] analysis. We view the analysis of Narasimhan et al. and that of Bredehoeft as being complementary to each other in the sense that the former requires an independent estimation of the tidal stress while the later requires an independent estimation of the tidal dilatation. In the absence of in situ measurements of tidal stress and dilatation, both approaches will require a certain amount of theoretical calculations based on reasonable parameters. Additional field measurements of pore pressure, tidal stress, and dilatation at experimental sites will undoubtedly improve our understanding of the response of well aquifer systems to earth tides.

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REFERENCES

Arditty, P. C., H. J. Ramey, Jr., and A. M. Nur, Response of a closed well reservoir system to stress induced by earth tides, paper presented at the 53rd Annual Meeting, Soc. Pet. Eng./Am. Inst. Min. Eng., Houston, Tex., Oct. 1978.

Beaumont, C., and J. Berger, An analysis of tidal strain observations from the United States of America, I, The laterally homogeneous tide, *Bull. Seismol. Soc. Am., 65*(6), 1613–1629, 1975.

Biot, M. A., General theory of three-dimensional consolidation, *J. Appl. Phys., 12*, 155–164, 1941.

Bodvarsson, G., Confined fluids as strain meters, *J. Geophys. Res., 75*(14), 2711–2718, 1970.

Bredehoeft, J. D., Response of well aquifer systems to earth tides, *J. Geophys. Res., 72*(12), 3075–3078, 1967.

Freeze, R. A., and J. A. Cherry, *Groundwater*, Prentice-Hall, Englewood Cliffs, N. J., 1979.

Jacob, C. E., On the flow of water in an elastic artesian aquifer, *Eos Trans. AGU, 27*, 574–586, 1940.

Marine, I. W., Water level fluctuations due to earth tides in a well pumping from slightly fractured crystalline rocks, *Water Resour. Res., 11*(1), 165–173, 1975.

Melchior, P., Die gezeiten in unterirdischen flussigkarten, *Erdoel Kohle, 13*, 312–317, 1960.

Narasimhan, T. N., and B. Y. Kanehiro, A note on the meaning of storage coefficient, *Water Resour. Res., 14*(2), 423–429, 1980.

Narasimhan, T. N., B. Y. Kanehiro, and P. A. Witherspoon, Interpretation of earth tide response of three deep, confined aquifers, *J. Geophys. Res., 89*(B3), 1913–1924, 1984.

Nur, A., and J. D. Byerlee, An exact effective stress law for elastic deformation of rocks with fluids, *J. Geophys. Res., 76*(26), 6414–6419, 1971.

Robinson, E. S., and R. T. Bell, Tides in confined well aquifer systems, *J. Geophys. Res., 76*(8), 1857–1869, 1971.

Van der Kamp, G., and J. E. Gale, Theory of earth tide and barometric effects in porous formations with compressible grains, *Water Resour. Res., 19*(2), 538–544, 1983.

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