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Determination of Fluid Flow Properties From the Response of Water Levels in Wells to Atmospheric Loading

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The water level in a well that taps a partially confined aquifer is often sensitive to atmospheric loading. The magnitude and character of this response is partly governed by the well radius, the lateral hydraulic diffusivity of the aquifer, the thickness and vertical pneumatic diffusivity of the unsaturated zone, and the thickness and vertical hydraulic diffusivity of the saturated zone overlying the aquifer. These key elements can be combined into five dimensionless parameters that partly govern the phase and attenuation of the response. In many cases, the response of a well to atmospheric loading can be broken up into a high-, intermediate-, and low-frequency response. The high-frequency response is governed largely by the well radius and lateral diffusivity of the aquifer. The intermediate-frequency response is governed by the loading efficiency of the aquifer. The low-frequency response is governed by the vertical pneumatic diffusivity and thickness of the unsaturated zone and the vertical hydraulic diffusivity and thickness of the saturated material above the aquifer. Cross-spectral estimation is used to fit the response to atmospheric loading of three water wells to the theoretical curves in order to yield estimates of three of the key dimensionless parameters. These estimates then are used to make estimates or place bounds on the vertical pneumatic diffusivity of the unsaturated zone, the lateral permeability of the aquifer, and the composite vertical hydraulic diffusivity of the overlying saturated materials.

INTRODUCTION

The water level in a well is often sensitive to atmospheric loading. Figure 1 compares a hydrograph of one of the wells to be examined in detail to local barometric pressure and tidal strain. The well responds inversely to barometric pressure changes, a phenomenon first rigorously examined by *Jacob* [1940]. The well also responds to tidal strains (compression is positive). If the aquifer is perfectly confined and has high lateral transmissivity or the well has a small diameter, the response of a water well to atmospheric loading and Earth tides will be a direct indication of the undrained response of the aquifer to imposed deformation. Under these conditions, changes in atmospheric pressure are related to changes in the water level of the well by a simple linear coefficient called the barometric efficiency [*Jacob*, 1940] or static-confined barometric efficiency (S. Rojstaczer and D. C. Agnew, The static response of the water level in an open well to areally extensive deformation under confined conditions, submitted to *Journal of Geophysical Research*, 1988) (hereafter referred to RA, 1988); changes in Earth-tide induced strain are related to changes in water level by a simple linear coefficient sometimes called the static-confined areal strain sensitivity (RA, 1988). If these coefficients are known or can be inferred, it is theoretically possible to determine the elastic properties and porosity of the aquifer [*Bredehoeft*, 1967; *Van der Kamp and Gale*, 1983; RA, 1988].

Aquifers, however, are never perfectly confined and their transmissivity can range in value over many orders of magnitude. Wells may be large in diameter. Hence the response of a water well to atmospheric loading and Earth tides may not always be a direct indication of the undrained, or static-confined, response of the aquifer. The response of aquifers to Earth tides and tectonic strain under conditions of partial drainage or partial confinement is discussed in a related paper [*Rojstaczer*, 1988]. The focus of this paper is on the response of water wells to atmospheric loading.

Figure 2(top) shows conceptually that air flow and groundwater flow can influence the response of a well to atmospheric loading. When atmospheric pressure changes slowly, air flow through the unsaturated zone and groundwater flow between the aquifer and the water table cause the aquifer response to be partially drained. When atmospheric pressure changes take place rapidly, aquifer response may be nearly undrained, but radial groundwater flow into and out of the well can strongly attenuate water well response if lateral aquifer transmissivity is low or well diameter is large. These deviations from the static-confined response cause the barometric efficiency of a well to be a function of the length of time or width of frequency band over which the atmospheric pressure change takes place.

It is instructive to examine the idealized response of the well-aquifer system shown in Figure 2(top) to a step change in atmospheric load ΔP. Initially, the aquifer and partial confining layer are pressurized instantaneously via grain to grain contact due to the change in surface load. The pressure is changed by an amount $\gamma''\Delta P$ in the confining layer and $\gamma\Delta P$ in the aquifer, where γ'' and γ are the loading efficiencies of the partial confining layer and the aquifer, respectively (RA, 1988). In contrast to the aquifer and the partial confining layer, the pressure change at the water surface of the open well is ΔP. The water level change at the water table, due to its high storage, is negligible. There are thus four imbalances in pressure potential due to the step change in atmospheric load that induce fluid flow: (1) vertical air flow induced by the pressure imbalance ΔP between the Earth's surface and the water table; (2) vertical groundwater flow induced by the pressure potential imbalance $\gamma''\Delta P$ between the water table and the confining layer; (3) vertical groundwater flow induced by the pressure potential imbalance $(\gamma'' - \gamma)\Delta P$ between the confining layer and the aquifer; and (4) lateral groundwater flow in-

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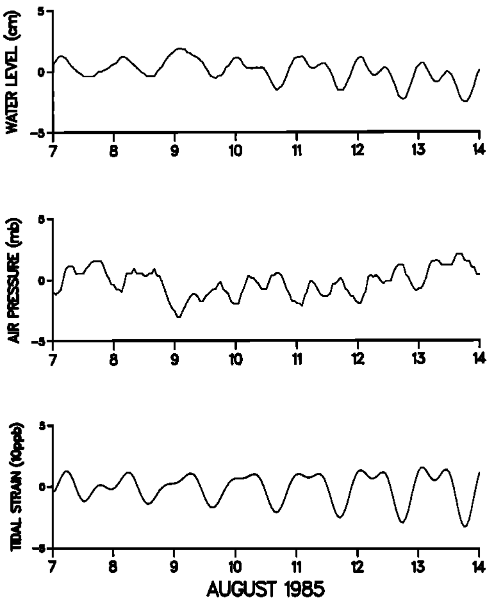

Fig. 1. Hydrograph at Well TF during the second week of August 1985 with corresponding barograph and theoretical tidal strain.

duced by the pressure potential imbalance $(1 - \gamma)\Delta P$ between the open water well and the aquifer.

All of these four imbalances induced by the step load will be established instantaneously. If the loading efficiencies γ and γ'' are nearly equal, then groundwater flow induced by the pressure imbalance $(\gamma''-\gamma)\Delta P$ will be negligible and we are left with three significant pressure potential imbalances. In this paper, I assume that γ'' equals γ. This essentially restricts the analysis to conditions where the confining layer and aquifer possess similar elastic properties and porosities or the aquifer is very thin and possesses a high vertical permeability relative to its lateral permeability.

The remaining three pressure imbalances caused by the step change in atmospheric load can (under certain conditions that will be examined below) cause water well response to occur in four distinct phases. The qualitative water well response to the step load is shown in Figure 2(middle). The qualitative pressure change in the unsaturated zone, partial confining layer and aquifer during each of the four phases is shown in Figure 2(bottom). Initially (phase 1), water flows out of the well into the aquifer driven by the pressure potential imbalance between the well and the aquifer. The water level in the well eventually drops by an amount $(1 - \gamma)\Delta P/\rho g$ at which point the well is in equilibrium with the undrained response of the aquifer (phase 2). The water well response temporarily forms a plateau whose width is governed by the length of time it takes for groundwater flow to the water table to influence the pressure of the aquifer.

If the unsaturated zone is thick or possesses low air permeability, the pressure potential at the water table does not change for a substantial period of time. The confining layer and eventually the aquifer, however, gradually depressurize due to groundwater flow to the water table and the water level in the well drops in response to this change in aquifer pressure potential (phase 3). The aquifer continues to depressurize and the water level in the well drops an additional $\gamma\Delta P/\rho g$ so that the total water level change is $\Delta P/\rho g$. Once air pressure begins to increase at the water table, however, a new pressure imbalance between the water table and the aquifer is created. Water moves back into the aquifer and partial confining layer. The water level in the well increases in response to this increase in aquifer pressure (phase 4) and eventually returns to its original static position once air pressure at the water table, the atmospheric load and the aquifer pressure are in static equilibrium.

Although examination of the response of a water well to step changes in deformation is useful for illustrative purposes, it is more quantitatively tractable to examine the response to periodic changes. Numerous studies have examined the response of wells to areally extensive deformation as a function of frequency. *Cooper et al.* [1965] and *Bodvarsson* [1970] examined theoretically the high-frequency response of water wells to deformation under the assumption that the aquifer was hydraulically isolated from the water table in the frequency range of interest. *Johnson* [1973] and A. G. Johnson and A. Nur (unpublished manuscript, 1978) examined the the-

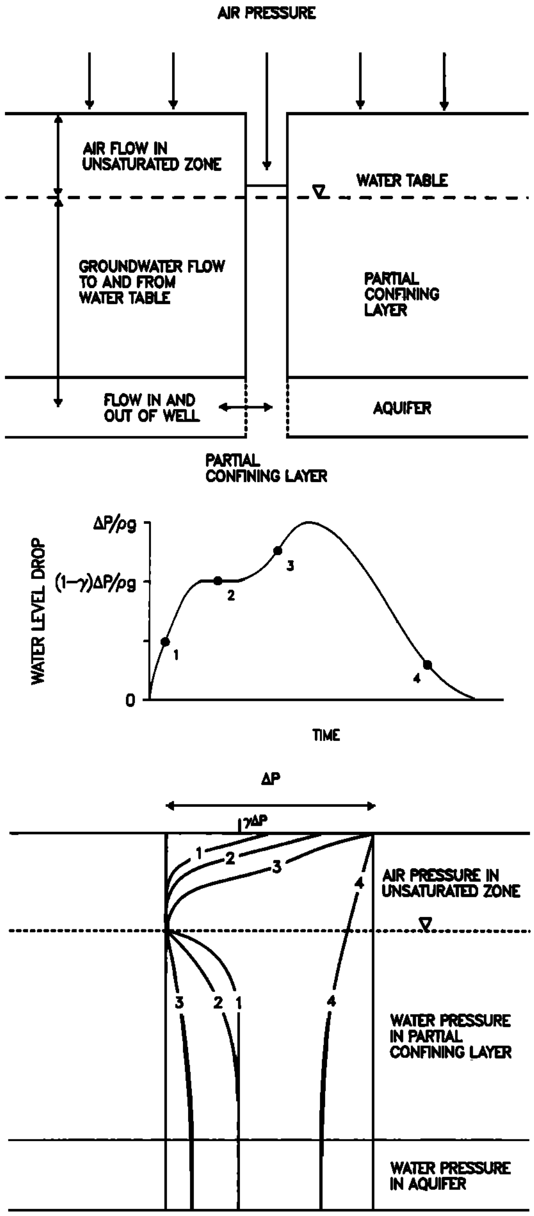

Fig. 2. (Top) Cross section of well responding to atmospheric loading and principal sources of attenuation and amplification of well response. (Middle) Idealized response of a well to a step change in atmospheric load. (Bottom) Profile of pressure response due to step change in atmospheric load at four time periods.

oretical response of water wells to deformation as a function of frequency under the assumptions that the unsaturated zone did not influence the response, inertial effects within the well were negligible, and the water table could be idealized as a spherically shaped boundary. *Yusa* [1969] and *Weeks* [1979] examined the response of water table wells to atmospheric loading due to the influence of the unsaturated zone under the assumption that the fluid pressure change at the water table was the average pressure change of the aquifer and that lateral transmissivity was high enough to allow for unattenuated groundwater flow between the aquifer and the borehole. *Morland and Donaldson* [1984], *Gieske* [1986], and *Hsieh et al.* [1987] have examined the response of water wells to deformation induced by Earth tides and/or atmospheric loading under the assumption that water table influences and inertial effects were negligible.

This study extends the results noted above by unifying many aspects of the different theoretical models. I examine theoretically the response of water wells to atmospheric loading by including the influences of groundwater flow between the borehole and the aquifer, groundwater flow between the aquifer and the water table, and air flow between the land surface and the water table through the unsaturated zone. I examine the theoretical response of water wells to atmospheric loading as a function of frequency under conditions where the well taps a partially confined aquifer. This theoretical model is applied to the response of three water wells to atmospheric loading inferred from cross-spectral estimation [e.g., *Bendat and Piersol*, 1986] to yield estimates of or place bounds on the following fluid flow parameters: pneumatic diffusivity of the unsaturated zone, vertical hydraulic diffusivity of the partial confining layer, and lateral permeability of the aquifer. It should be noted that the results shown here have many similarities to the response of wells tapping water table aquifers (S. Rojstaczer and F. Riley, The influence of vertical fluid flow on the response of the water level in a well to atmospheric loading under unconfined conditions, submitted to *Water Resources Research*, 1988).

THEORETICAL RESPONSE OF WELLS IN PARTIALLY CONFINED AQUIFERS TO PERIODIC ATMOSPHERIC LOADING

The response of a water well to atmospheric loading can be conveniently broken up into five processes: (1) mechanical loading of the aquifer due to the surface load; (2) pressurization at the water surface of the open well due to the air load; (3) flow of air between the Earth's surface and the water table; (4) flow of groundwater between the water table and the aquifer; and (5) flow of groundwater between the aquifer and the borehole. In order to make the analysis analytically tractable I make some simplifying assumptions about these processes. I assume that the undrained response of the aquifer and the partial confining layer to surface loading are the same; this essentially assumes that the compressibility, porosity, and Poisson's ratio are vertically and laterally uniform. I make the assumption that air flow between the Earth's surface and the water table and groundwater flow in the partial confining layer, owing to the great lateral extent of the atmospheric load, are vertical. I also make the assumption, common to the analysis of partially confined aquifers [*Hantush*, 1955, 1960; *Neuman and Witherspoon*, 1969*a*], that groundwater flow between the aquifer and the borehole is horizontal. These assumptions allow me to uncouple the three-dimensional nature of the problem into three flow problems, two of which have a strictly vertical component of flow and one of which has a strictly radial component of flow in the aquifer and vertical component of flow within the partial confining layer: (1) vertical air flow between the Earth's surface and the water table; (2) vertical groundwater flow between the water table and the aquifer; and (3) horizontal groundwater flow between the aquifer and the borehole with concomitant "leakance" [*Jacob*, 1946] from the overlying partial confining layer.

Vertical Air Flow Between the Earth's Surface and the Water Table

Periodic vertical flow of air between the Earth's surface and the water table is governed by a simple diffusion equation [*Buckingham*, 1904; *Weeks*, 1979]:

$$D_a\, \partial^2 p_a/\partial z^2 = \partial p_a/\partial t \tag{1}$$

subject to the following boundary conditions:

$$p_a(-L, t) = A \cos(\omega t) \tag{2a}$$

$$p_a(L, t) = A \cos(\omega t) \tag{2b}$$

where p_a is the air pressure, D_a is the pneumatic diffusivity, and A and ω are the amplitude and frequency, respectively, of the pressure wave. The boundary $-L$ is taken to be the Earth's surface, the water table is at a depth of 0, and the zone from depth 0 to depth L is an artifice to assure that at the water table there is no air flux. The solution for air pressure at the water table ($z = 0$) is given by [*Rojstaczer*, 1988]

$$p_a = (M - iN)A \exp(i\omega t) \tag{3}$$

where M and N are

$$M = \frac{2\cosh(\sqrt{R})\cos(\sqrt{R})}{\cosh(2\sqrt{R}) + \cos(2\sqrt{R})} \tag{4a}$$

$$N = \frac{2\sinh(\sqrt{R})\sin(\sqrt{R})}{\cosh(2\sqrt{R}) + \cos(2\sqrt{R})} \tag{4b}$$

and R is a dimensionless frequency referenced to the pneumatic diffusivity D_a and the depth L from the Earth's surface to the water table:

$$R = L^2\omega/2D_a \tag{5}$$

Carslaw and Jaeger [1958, p. 105] give the solution of (1) subject to the boundary conditions of (2) strictly in terms of phase and gain.

It should be noted that the inverse of the dimensionless frequency R is analogous to the dimensionless time $1/u$ well-known in well hydraulics. The difference is that time has been replaced by frequency, the diffusivity of the aquifer has been replaced by the pneumatic diffusivity of the unsaturated zone and the radial distance from the well has been replaced by the thickness of the unsaturated zone.

Vertical Groundwater Flow Between the Water Table and the Aquifer

Groundwater flow between the water table and the aquifer under partially confined conditions is assumed to be strictly vertical and occurs strictly within the partial confining layer overlying the aquifer. Taking compression to be positive, the governing equation for pore pressure response due to periodic atmospheric loading is (RA, 1988)

$$D\, \partial^2 p/\partial z^2 = \partial p/\partial t + \omega\gamma A \sin \omega t \tag{6}$$

where D is the vertical hydraulic diffusivity of the partial confining layer under conditions where the principal components of horizontal strain are 1/2 the vertical strain, p is the pore pressure change in excess of hydrostatic, and γ is the loading efficiency. The loading efficiency is the ratio of change in pore pressure to change in surface load under undrained conditions. The loading efficiency γ used here is qualitatively the same as that given by *Van der Kamp and Gale* [1983] and the tidal efficiency given by *Jacob* [1940]. As noted elsewhere (RA, 1988), the major difference is that the γ used here incorporates the influence of horizontal deformation. The source term in (6) is due to the essentially instantaneous transmission of the surface load via grain to grain contact to the subsurface.

If I take compressive stresses to be positive, the appropriate boundary conditions are

$$p(0, t) = MA \cos(\omega t) + NA \sin(\omega t) \quad (7a)$$

$$p(\infty, t) = A\gamma \cos(\omega t) \quad (7b)$$

where I again take $z = 0$ to be the water table. The water table boundary condition is the solution of (3). The solution of (6) subject to boundary conditions given in (7) is [*Rojstaczer*, 1988]:

$$p = (M + iN - \gamma)A \exp(-(i + 1)(0.5qS')^{1/2}) \cdot \exp(i\omega t) + A\gamma \exp(i\omega t) \quad (8)$$

where S' is the storage coefficient of the confining layer under conditions of surface loading and q is a dimensionless frequency referenced to the vertical hydraulic conductivity of the partial confining layer K' and the distance b' between the water table and the top of the aquifer (i.e., the thickness of the partial confining layer):

$$q = b'\omega/K' \quad (9)$$

It should be noted that the term $0.5qS'$ is the dimensionless frequency Q used in a later section and defined as

$$Q = qS'/2 = b'^2\omega/2D \quad (10)$$

where D is the vertical hydraulic diffusivity (see equation (6)) of the partial confining layer under conditions of surface loading.

Flow Between the Borehole and the Aquifer

Groundwater flow between the borehole and the aquifer is driven by the difference between the water level in the well and the aquifer pressure in terms of head. Flow within the aquifer, as previously noted, is assumed to be strictly horizontal and the influence of the partial confining layer is described by a leakance term. Under these conditions, the governing equation is [*Jacob*, 1946]

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} - \frac{K's}{Kbb'} = \frac{S_s \partial s}{K \partial t} \quad (11)$$

subject to the following boundary conditions [*Cooper et al.*, 1965]:

$$s(\infty, t) = 0 \quad (12a)$$

$$\lim_{r \to 0} \frac{r\partial s}{\partial r} = \frac{\omega r_w^2 x_0}{2Kb} \sin \omega t \quad (12b)$$

where s is the drawdown within the aquifer caused by a periodic volumetric discharge within the well, K is the hydraulic conductivity of the aquifer, b is the thickness of the aquifer, S_s is the specific storage of the aquifer under conditions of no horizontal deformation, r_w is the radius of the well, and x_0 is the amplitude of the water level fluctuation within the well casing produced by the volumetric discharge. This periodic steady state problem is solved in the appendix. The solution for the drawdown at the well just outside the well screen s_w is

$$s_w = i0.5Wx_0K_0\{[W^2(S^2 + 1/q^2)]^{0.25} \cdot \exp[i0.5\{\tan^{-1}(qS)\}]\} \exp(i\omega t) \quad (13)$$

where K_0 is the modified Bessel function of the second kind of order zero [*Olver*, 1972; *Tranter*, 1968], S is simply $S_s b$, the storage coefficient of the aquifer and W is

$$W = \omega r_w^2/Kb \quad (14)$$

It should be noted that W is a dimensionless frequency (analogous to the inverse of dimensionless time used in well hydraulics) and $1/q$ is the conventional leakance of well hydraulics divided by frequency.

The solution given by (13) assumes that (1) the water table does not change in response to periodic discharge from the well; (2) the partial confining layer has negligible specific storage; (3) pore pressure changes induced by the fluctuating water level induce only vertical deformation; and (4) the well is a line source. In essence (13) is the same solution given by *Hantush and Jacob* [1955] for aquifer response to pumpage under conditions of leakance; the difference is that the well discharges at a periodic rate rather than at a constant rate. *Neuman and Witherspoon* [1969*b*] have examined the error involved in assumptions 1 and 2. Their results indicate that confining layer specific storage and changes in water table height can be ignored when the dimensionless parameter $(W/q)^{1/2}$ and a dimensionless parameter β are less than 0.01 where β is defined as

$$\beta = r_w/4b'(K'S_s'/KS_s)^{1/2} \quad (15)$$

In (15), S_s' is the specific storage of the confining layer under conditions of no horizontal deformation. Since confining layer permeabilities will not be greater than aquifer permeabilities and the well radius will be significantly less than the thickness of the confining layer and the aquifer, the dimensionless terms $(W/q)^{1/2}$ and β will almost always be less than 0.01. These results indicate that changes in water table height do not significantly influence aquifer response and that the specific storage of the partial confining layer, although it does influence vertical flow (see equation (8)), does not significantly influence horizontal flow in the aquifer.

The assumption that pore pressure changes induced by well dicharge do not induce horizontal deformation is a standard assumption in groundwater hydraulics. *Gambolati* [1974, 1977] examined the error in this assumption and found that (in the absence of leakance) drawdown accompanying well discharge is not significantly influenced by horizontal deformation when the well taps an aquifer whose thickness is less than 1/2 its average depth.

Response of a Well to Atmospheric Loading: General Case

The response of a well to atmospheric loading can be obtained, in the absence of inertial effects, by combining the solutions given in (8) and (13). Since we are concerned only with slowly varying water level fluctuations, inertial effects in

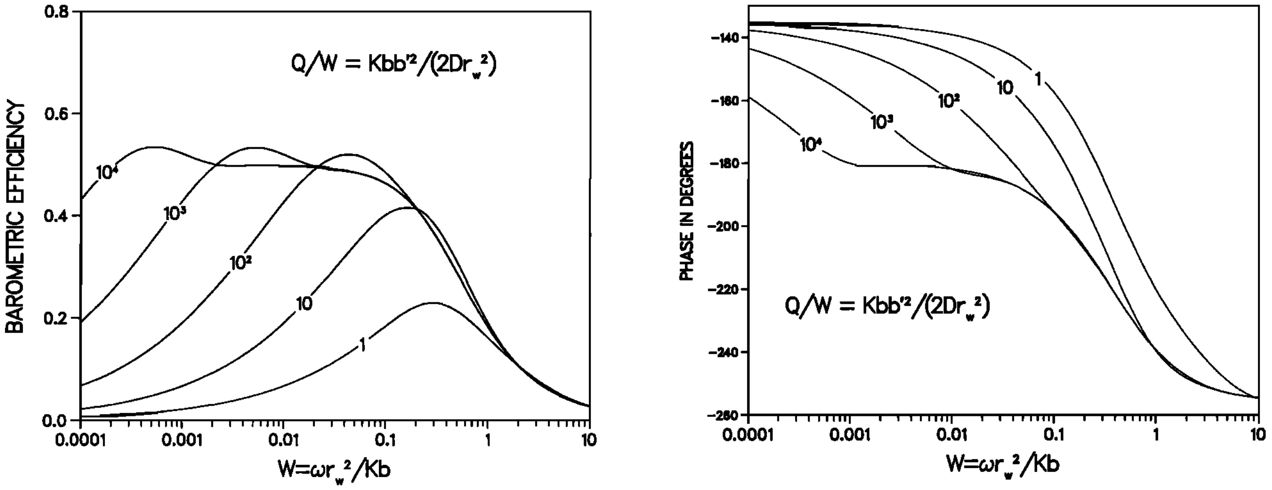

Fig. 3. (Left) Barometric efficiency and (right) phase of response of a well to atmospheric loading as a function of Q/W when S and S' equal 0.0001. Static-confined barometric efficiency $(1-\gamma)$ is 0.5 and $R \ll Q$.

the borehole can be ignored and the relation between the amplitude of the water level fluctuation in the well, x_0 (measured positive upwards), and the amplitude of the atmospheric load A is

$$x_0 = -A/\rho g + p_0/\rho g - s_0 \tag{16}$$

where p_0 is the far field pore pressure of the aquifer (pore pressure at a radial distance where the influence of the well is negligible) p, divided by exp $(i\omega t)$, and s_0 is the drawdown at the well s_w, divided by exp (iwt)

$$p_0 = p \exp(-i\omega t) \tag{17a}$$

$$s_0 = s_w \exp(-i\omega t) \tag{17b}$$

Equation (16) describes the response of the well in the frequency domain and states that the change in water level in the well plus the atmospheric load (in terms of equivalent change in water level) equals the far field pore pressure (in terms of equivalent water level) minus the drawdown at the well.

It is useful to write (16) in terms of the gain or barometric efficiency BE and the phase θ of the response

$$BE(\omega) = \left|\frac{x_0 \rho g}{A}\right| = \left|\frac{p_0 - A - s_0 \rho g}{A}\right| \tag{18a}$$

$$\theta(\omega) = \arg(x_0 \rho g / A) \tag{18b}$$

where the brackets in (18a) denote the modulus of the complex function and arg in (18b) denotes the inverse tangent of the ratio of the imaginary component to the real component of the complex function. Equation (18a) describes the ratio of the amplitude of the water level fluctuation to the amplitude of the atmospheric load (in terms of equivalent water level). Equation (18b) describes the phase shift between the atmospheric load wave and the water level fluctuation. Under conditions where the confining layer has zero permeability and the aquifer transmissivity is high, p_0 would be equal to $A\gamma$ and the barometric efficiency BE would simply be one minus the loading efficiency γ. The phase shift would be a flat $-180°$ for all observable frequencies of the atmospheric loading wave. However, under conditions where the confining layer has a finite permeability and the aquifer transmissivity is low, both the barometric efficiency and the phase will be a strong function of frequency.

In this study, barometric efficiency depends on frequency. The value for efficiency that reflects the undrained response of the aquifer $(1-\gamma)$ is termed the static-confined barometric efficiency. Equations (8), (13), and (18) indicate that the barometric efficiency BE and phase θ of the response are a function of six dimensionless parameters: (1) R, the dimensionless unsaturated zone frequency; (2) q, the dimensionless confining layer frequency; (3) S', the storage of the confining layer; (4) S, the storage of the aquifer; (5) γ, the loading efficiency of the partial confining layer and aquifer; and (6) W, the dimensionless aquifer frequency.

The barometric efficiency and phase of the response of the water well are shown in Figure 3 as a function of dimensionless aquifer frequency W and the ratio of dimensionless confining layer frequency $qS'/2$ or Q, to W. In Figure 3, R is assumed to be much less than Q ($R/Q = 0.0001$), S and S' are 0.0001, and the static-confined barometric efficiency of the aquifer is 0.5. These constraints allow us to examine water well response under conditions where the aquifer has typical elastic properties, and unsaturated zone effects, due either to a shallow water table or a high pneumatic diffusivity, are negligible. The assumption of negligible unsaturated zone effects will be relaxed in a subsequent section. The dimensionless ratio Q/W is a measure of the frequency above which there is significant attenuation and phase shift due to limited groundwater flow between the borehole and the aquifer relative to the frequency below which water table drainage significantly influences aquifer response. When Q/W is large, a frequency band exists over which there is little attenuation and phase shift in water well response. When Q/W is small, we can expect that the water well response will show significant attenuation and phase shift (relative to $-180°$) for all frequencies. Because unsaturated zone effects have been neglected, the response shown is qualitatively similar to the theoretical response given by *Johnson* [1973] and A. G. Johnson and A. Nur (unpublished manuscript, 1978); the major difference between this set of theoretical curves and their results is due to their approximation that the water table is a spherically shaped boundary which encloses a spherically shaped aquifer.

For values of Q/W much less than 1000, the static-confined barometric efficiency is never observed; barometric response is attenuated with concomitant phase shift throughout the entire frequency range. Physically, values of Q/W less than 100 indi-

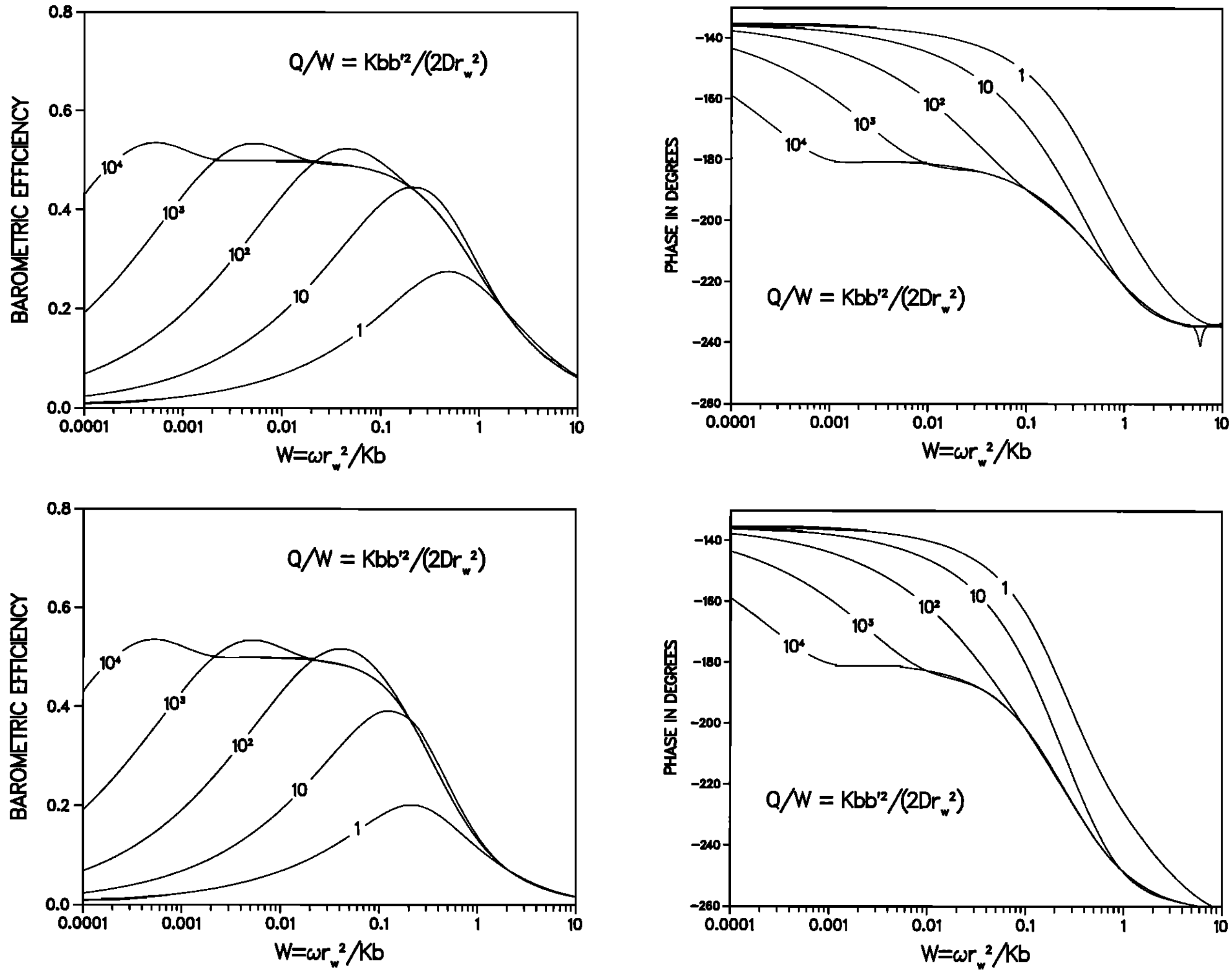

Fig. 4. (Top left) Barometric efficiency and (top right) phase of response of a well to atmospheric loading as a function of Q/W when S and S' equal 0.01. (Bottom left) Barometric efficiency and (bottom right) phase when S and S' equal 1×10^{-6}. Static-confined barometric efficiency $(1 - \gamma)$ is 0.5.

cate conditions where the water table has a strong influence on water well response over a wide frequency band; the aquifer becomes isolated from water table influences only when frequencies are so high, relative to aquifer transmissivity, that limited groundwater flow between the aquifer and the borehole cause significant attenuation of response.

For values of Q/W greater than 1000, three distinct stages of response can be observed: an intermediate-frequency response, a low-frequency response, and a high-frequency response. At intermediate frequencies, air pressure response forms a plateau in both phase and barometric efficiency that increases in width with increasing values of Q/W. This response is analogous to the response to a step load during stage 2 (see Figure 2). In this frequency band, the static-confined barometric efficiency is observed and there is little phase shift between the atmospheric pressure wave and the water well response (the phase shift of $-180°$ is due to the inverse relation between water level and atmospheric pressure). Physically, water table influences are negligible in this frequency band and the aquifer transmissivity is high enough to allow for well response to be unattenuated.

It should be noted that for frequencies overlapping the low- and intermediate-frequency bands, barometric response slightly exceeds the static-confined barometric efficiency. There is no analog to this slight amplification in the response of a water well to step changes in atmospheric load. The amplification of response is due to resonance: the influence of the water table is slight, but it has a phase shift that weakly reinforces the nearly confined water well response.

In the low-frequency band, the response is distinguished by increasing attenuation and phase advance with decreasing frequency. This response is analogous to stage 4 in Figure 2: as frequency decreases, water table influences become more significant and the response asymptotically approaches 0. It should be noted that stage 3 noted in Figure 2 (barometric efficiency achieving a value of 1 due to early water table influences) does not appear in Figure 3. This is because unsaturated zone effects are assumed to be negligible.

In the high-frequency band, the response is characterized by increasing attenuation and phase lag with increasing frequency. This response is analogous to stage 1 in Figure 2. At these frequencies, aquifer transmissivity is low enough (for the given well bore storage) to limit groundwater flow between the aquifer and the borehole and as frequency increases, the response asymptotically approaches 0.

Figure 4 shows the influence that the storage of the confining layer and aquifer have on the response. In Figures 4(top left) and 4(top right) storage for both the confining layer and the aquifer are 0.01; in Figure 4(bottom left) and 4(bottom right) they are 1×10^{-6}. Both sets of response curves are qualitatively similar to the response curves in Figure 3. As in Figure 3, the response can be compartmentalized into three frequency bands for values of Q/W greater than 1000. At low frequencies, the sensitivity to storage is negligible for a fixed value of Q/W. This lack of sensitivity is due to the minor amount of well drawdown at low frequencies. At high frequencies, decreasing storage causes greater attenuation and phase shift, a phenomenon that will be considered in detail in the following section.

It is useful to determine, given typical aquifer and confining layer properties and geometries, whether the parameter Q/W can realistically have a value greater than 1000. Given an aquifer thickness of 30 m and hydraulic conductivity of 2×10^{-7} m/s, a confining layer hydraulic conductivity of 10^{-9} m/s and specific storage 3×10^{-6} m^{-1}, and a well radius of 0.1 m, the dimensionless parameter Q/W has a value of approximately $1 \times b'^2$, where b' is in meters. For Q/W to exceed 1000 under these conditions, confining layer thickness must be in excess of 30 m. This result indicates that in many instances the parameter Q/W will be greater than 1000 and water well response can be broken up into three distinct frequency bands. In the following sections, I examine the high-frequency band and low-frequency band in detail.

High-Frequency Response

In the high-frequency band, the well is isolated from water table and unsaturated zone influences. As a result, aquifer pressure p_0 is a constant and the dimensionless frequency q is effectively infinite. The barometric efficiency and phase of the response are described by

$$BE(\omega) = \left| \frac{A(\gamma - 1) - s_0 \rho g}{A} \right| \quad (19a)$$

$$\theta(\omega) = \tan^{-1} [\mathrm{Im}[A(\gamma - 1) - s_0 \rho g]/\mathrm{Re}[A(\gamma - 1) - s_0 \rho g] \quad (19b)$$

where Im and Re denote the imaginary and real parts of the function, respectively. Since aquifer pressure is related to the amplitude of the pressure wave by a constant γ water well attenuation and phase shift depend on only two out of the six dimensionless parameters: W and S. Of these two parameters, only the dimensionless aquifer frequency W strongly influences response. Figure 5 shows the barometric efficiency and phase of the water well response as a function of W and S. Because water table influences are negligible, the solution given here is nearly identical to the solution given by *Cooper et al.* [1965] for the steady state response of a well that taps a confined aquifer to periodic deformation at frequencies where inertial effects are insignificant. The only differences are that the phase has been shifted by $-180°$ due to the inverse relation between air pressure and water level and the amplitude of the response has been multiplied by the static-confined barometric efficiency $(1 - \gamma)$. As noted by *Hsieh et al.* [1987], the solution given by Cooper et al. indicates that the phase is only weakly dependent on aquifer storage, with less phase lag and attenuation slightly favored by high values of aquifer storage.

For all values of aquifer storage coefficient S large attenuation and phase shift occur only after dimensionless frequency W exceeds a value of 0.1. Thus the absence of any observable attenuation and phase shift with increasing frequency in a well's response places a lower bound on aquifer transmissivity if the radius of the well is known and the influence of the water table is slight in the frequency band of interest.

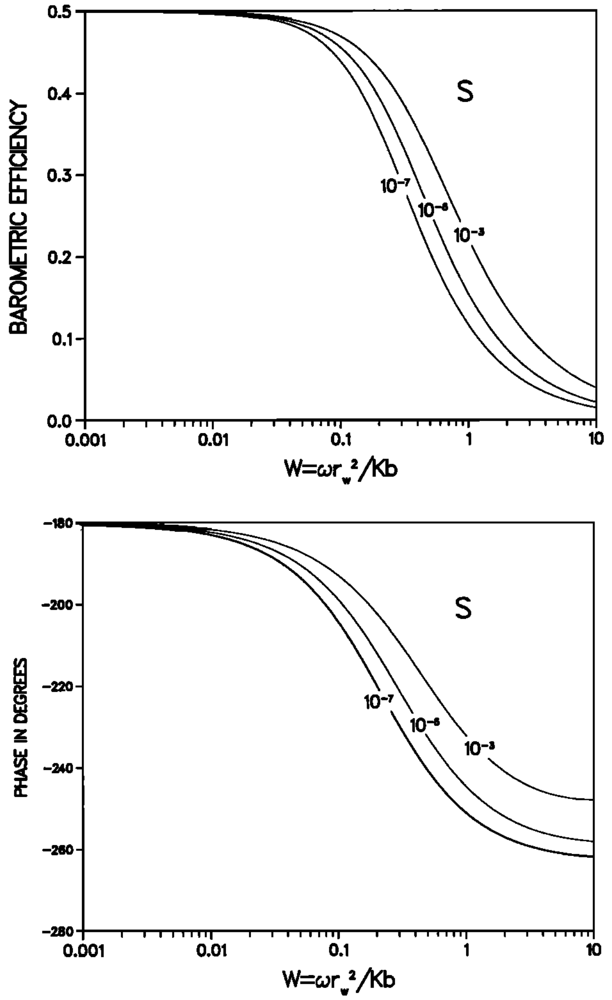

Fig. 5. High-frequency response in terms of barometric (top) efficiency and (bottom) phase as a function of S. Static-confined barometric efficiency $(1 - \gamma)$ is 0.5.

Low-Frequency Response

In the low-frequency band, the well is in equilibrium with aquifer pressure and the well drawdown s_0 can be assumed to be zero. The barometric efficiency and phase are described by

$$BE(\omega) = |p_0/A - 1| \quad (20a)$$

$$\theta(\omega) = \tan^{-1} (\mathrm{Im}(p_0/A - 1)/\mathrm{Re}(p_0/A - 1)) \quad (20b)$$

Since barometric efficiency and phase are strictly a function of aquifer pressure p_0 water well response is dependent on only three of the dimensionless parameters: γ (one minus the static-confined barometric efficiency), Q, and R.

Figure 6 shows the response of a water well in the low-frequency band as a function of dimensionless confining layer frequency Q and dimensionless unsaturated zone frequency R. The static-confined barometric efficiency is 0.5. The solution shown in the figure is essentially identical to a solution discussed elsewhere [*Rojstaczer*, 1988]; the only difference is that following hydrologic convention, compression is defined as positive.

In summary, water well response in the low-frequency band is a strong function of both R and Q. When the ratio R/Q is

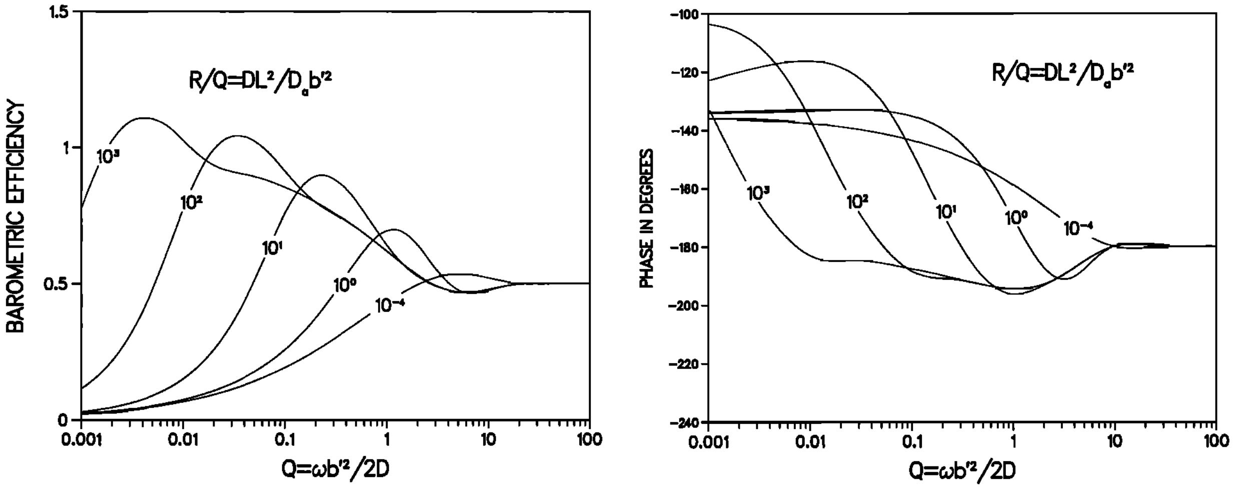

Fig. 6. Low-frequency response in terms of barometric (left) efficiency and (right) phase as a function of R/Q. Static-confined barometric efficiency $(1 - \gamma)$ is 0.5.

less than 10^{-4} the unsaturated zone has little influence on response and the barometric efficiency, which exhibits slight resonance at the high end of the frequency band, generally attenuates with decreasing frequency; the phase shows a nearly monotonic phase advance with decreasing frequency. For large values of R/Q, however, barometric efficiency exceeds the static-confined response over much of the frequency band analyzed. The increasing barometric efficiency with decreasing frequency is analogous to stage 3 in Figure 2. As previously noted, the response is caused by water table influence under conditions where the water table is strongly isolated from air pressure changes at the surface. For large values of R/Q, the phase lags slightly behind the air pressure over much of this frequency band.

Figure 7 shows the influence of the loading efficiency γ on well response. For aquifers with a loading efficiency of 0.20 (static-confined barometric efficiency of 0.80), the amplitude of the response is considerably higher than that shown in Figure 6 (static-confined barometric efficiency and γ equal 0.50), at dimensionless frequencies less than 1. The phase, in comparison to Figure 6, shows little in the way of a phase lag. For aquifers with a loading efficiency of 0.80 (static-confined barometric efficiency of 0.20), the amplitude of the response is considerably lower at dimensionless frequencies greater than 1. The phase, when R/Q is small, has a wide frequency band of significant phase lag.

Application of Theoretical Response

The above results indicate that water well response to atmospheric loading will be strongly dependent on the three dimensionless fluid flow parameters: R, Q, and W. If the response of a well can be fit to the theoretical solutions, it is possible to make estimates or place bounds on these three key parameters. Once these dimensionless parameters are estimated, it is then possible to make estimates of or place bounds on the fluid flow parameters that govern water well response: pneumatic diffusivity of the unsaturated zone, confining layer hydraulic diffusivity and aquifer permeability. The process of fitting well response as a function of frequency to dimensionless theoretical curves is analogous to the standard practice of fitting water level declines as a function of time in response to pumpage to "type curve" plots. The essential difference is that because the solutions given here are a function of frequency, there are two type curves that are fit simultaneously: one for barometric efficiency and one for phase.

In order to compare a water well's response to the theoretical solutions, we need to determine its transfer function or barometric efficiency and phase as a function of frequency. The transfer function that relates atmospheric loading to water level can be found using cross-spectral estimation [e.g., *Bendat and Piersol*, 1986]. For the water well records examined here, the transfer functions were obtained by (1) determining the power spectra and cross spectra for the water well record, the local atmospheric pressure record and the theoretical areal strain produced by the Earth tides, and (2) solving the following system of complex linear equations for every frequency:

$$\begin{vmatrix} BB & BT \\ TB & TT \end{vmatrix} \begin{vmatrix} HB \\ HT \end{vmatrix} = \begin{vmatrix} BW \\ TW \end{vmatrix} \tag{21}$$

where BB and TT denote the power spectra of the atmospheric pressure and Earth tides, respectively, BT and TB denote the cross spectrum and complex conjugate of the cross spectrum, respectively, between atmospheric loading and Earth tides, BW and TW denote the cross spectra between atmospheric loading and water level and Earth tides and water level, respectively, and HB and HT denote the transfer function between water level and atmospheric loading and water level and Earth tides, respectively. The Earth tides were included in the analysis because they have a strong influence on the response of the wells examined at diurnal and semidiurnal frequencies. Further details on how the transfer functions were determined are given elsewhere [*Rojstaczer*, 1988]. In the analysis below, the transfer functions were fit to the type curves by hand.

A description of the wells examined in this paper is given in Table 1. Two of these wells, TF and JC, are located near Parkfield, California and the other well is located near Mammoth Lakes, California. The aquifer permeabilities given in Table 1 were determined from specific capacity data (TF) or slug tests (JC, SC2). The aquifer permeabilities inferred from the slug tests as well as the thicknesses of the partial confining layers (depth from the water table to the top of the aquifer) at these wells indicate that the dimensionless ratio Q/W may be quite large; as a result, well response may take place in the three distinct bands noted above.

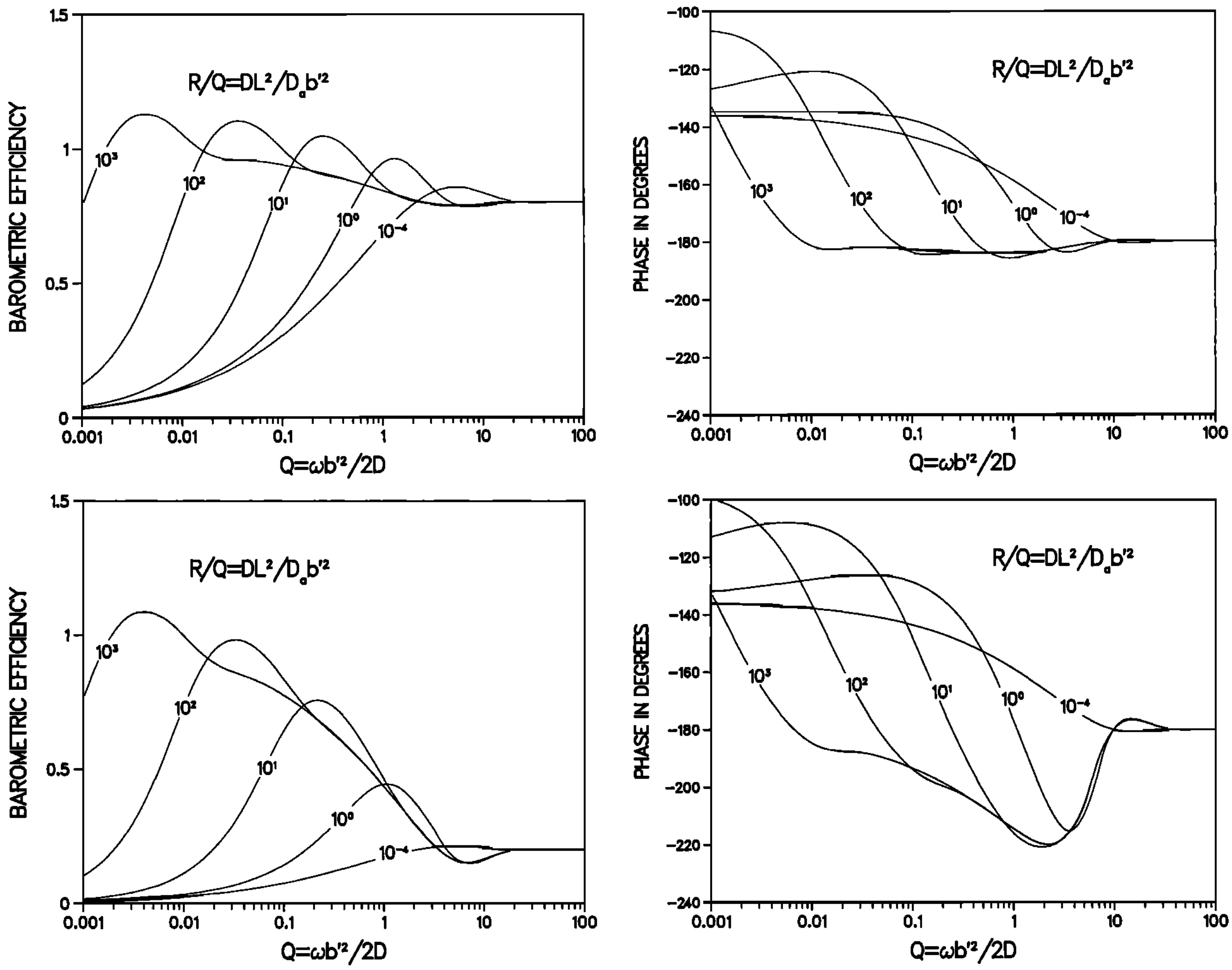

Fig. 7. Effect of surface loading efficiency on low-frequency response. (Top left) Barometric efficiency and (top right) phase when the loading efficiency γ is 0.2. (Bottom left) Barometric efficiency and (bottom right) phase when the loading efficiency γ is 0.8.

It is likely, however, that only a part of the complete theoretical response will be observed in any one well. The limited length of the data sets (about 150 days) and the lack of any large air pressure signal at frequencies greater than 2 cycles/day limit the band width over which we can estimate well response. For the wells examined here, we can obtain useful estimates of water well response in the frequency band of roughly 0.02 to 2 cycles/day. This band is only 2/5 of the frequency band detailed in Figure 3 and as a result, it is unlikely that the low, intermediate-, and high-frequency response can all be observed. In the well responses examined below, only the low- and intermediate-frequency responses are observed. The lack of a high-frequency response does serve, however, to place a lower bound on the aquifer permeabilities for these wells.

Well TF

The transfer function for the response of well TF to atmospheric loading is shown in Figure 8. Barometric efficiency peaks at 0.6 at a frequency of about 0.5 cycles/day. The phase which lags the atmospheric pressure at a frequency of 1 cycle/day begins to cross over and show phase advance with decreasing frequency at about 0.6 cycles/day. The figure also shows the model fit to the observed transfer function. The theoretical model indicates that the response in the frequency band of 0.02–2 cycles/day is dominated by water table influences. The confined response indicated by the model is only approached at the high end of the observed frequency band. The key parameters indicated by the model are a static-confined barometric efficiency of 0.37 and a value for both

TABLE 1. Description of Wells

Well Id.	Permeability, millidarcies	Open Interval, m	Depth to Water Table, m	Casing Diameter, m	Aquifer Lithology	Partial Confining Layer Lithology
TF	2×10^1	152–177	18	0.10	marine sediments	marine sediments
JC	5×10^1	147–153	14	0.10	diatomaceous sandstone	largely fine to medium grained sandstone
SC2	2×10^7	66–70	32	0.10	fractured basalt	basalt and glacial till

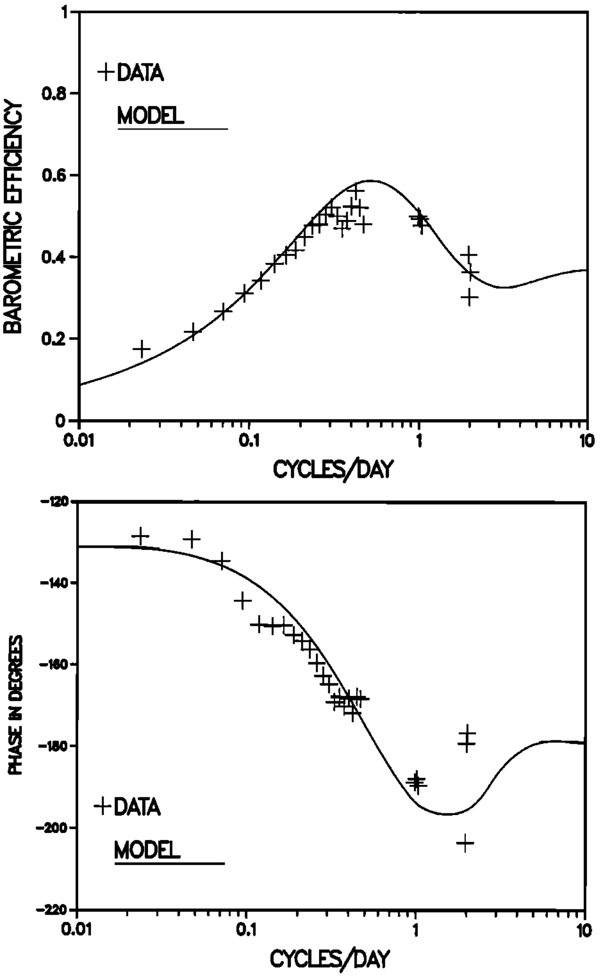

Fig. 8. Response of TF to atmospheric pressure in terms of (top) barometric efficiency and (bottom) phase. Fit to data is solid line denoted as Model.

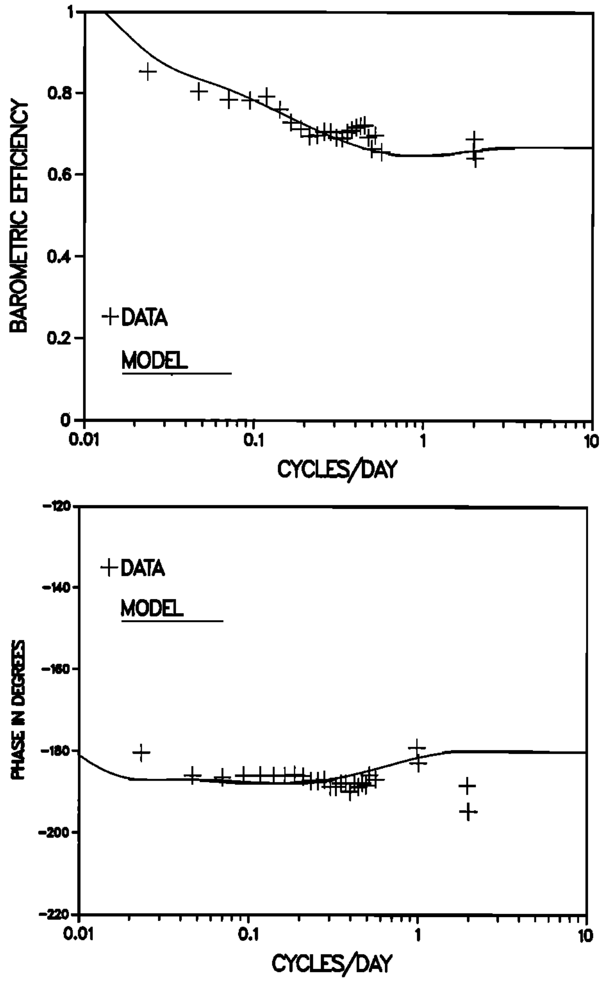

Fig. 9. Response of JC to atmospheric pressure in terms of (top) barometric efficiency and (bottom) phase. Fit to data is solid line denoted as Model.

dimensionless frequencies R and Q of 0.34ω where frequency is in terms of radians per day. The hydraulic and pneumatic diffusivities estimated from these values of R and Q are shown in Table 2. The specific storage for the aquifer under conditions of atmospheric loading is considered elsewhere (RA, 1988) and is determined from the inferred static-confined barometric efficiency and areal strain sensitivity for the well. It is estimated to be 2.2×10^{-6} m^{-1}. Assuming that the specific storage of the confining layer is close to that of the aquifer, I can obtain an estimate of the vertical permeability of the confining layer. This permeability is 10 mdarcy, a value slightly less than the permeability of the aquifer of 20 mdarcy indicated by the specific capacity data. The lack of any observable response that can be attributed to limited groundwater flow

TABLE 2. Estimate of Fluid Flow Properties of Wells

Well Id.	Aquifer Permeability, millidarcies	Confining Layer Hydraulic Diffusivity, cm²/s	Unsaturated Zone Pneumatic Diffusivity, cm²/s
TF	>10	5×10^2	9×10^0
JC	>60	2×10^2	2×10^{-2}
SC2	>90	1×10^0	$>1 \times 10^1$

Estimates for SC2 are from model 2 in Figure 10.

between the borehole and the aquifer places a lower bound on aquifer permeability. Assuming that the dimensionless frequency W is less than 0.1, the permeability of the aquifer is greater than 10 mdarcy, a value consistent with the specific capacity data.

Well JC

Figure 9 shows the transfer function for the well response at JC. Barometric efficiency shows a nearly monotonic change with decreasing frequency over the entire observed frequency band. The phase is nearly flat over the observed frequency band and indicates that the water level in the well lags slightly behind the atmospheric load. The fit to the theoretical model indicates that water well response is strongly governed by limited air flow between the Earth's surface and the water table. Like the response at TF, the static-confined response is approached at a frequency of 2 cycles/day. The inferred static-confined barometric efficiency determined from the model is 0.67. The dimensionless parameters R and Q are 100 and 1.0ω, respectively. The pneumatic and hydraulic diffusivities estimated from these parameters are shown in Table 2. The estimated hydraulic diffusivity of the partial confining layer is on the same order as that estimated at TF; the estimated pneumatic diffusivity is over two orders of magnitude less than that at TF. It should be noted that it is difficult to explain this

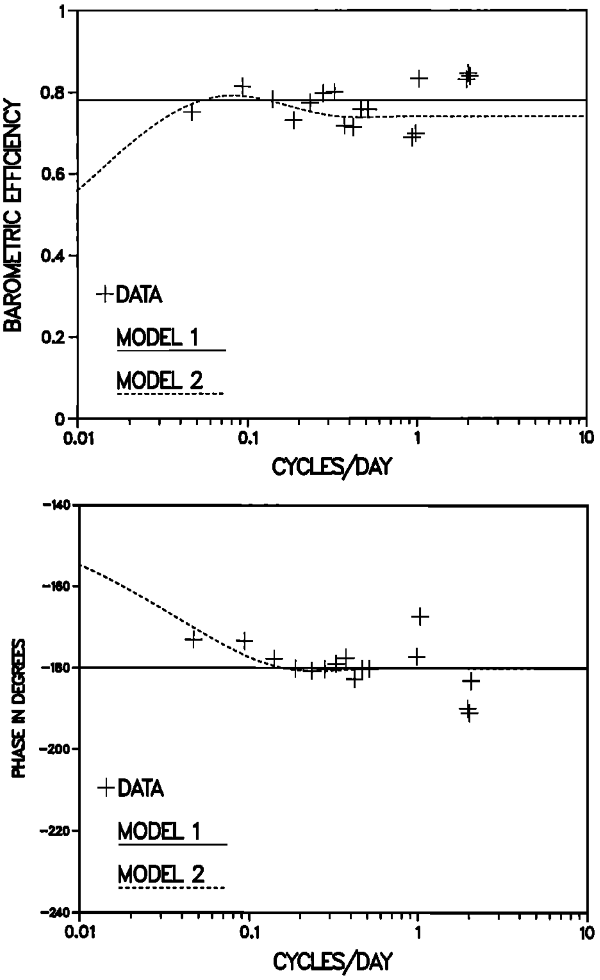

Fig. 10. Response of SC2 to atmospheric pressure in terms of (top) barometric efficiency and (bottom) phase. Fit to data are lines denoted as models 1 and 2.

difference on the basis of differences in site lithology. The specific storage of the aquifer is estimated elsewhere (RA, 1988) to be 2.3×10^{-6} m^{-1}. If the specific storage of the confining layer is close to that of the aquifer, the vertical permeability of the confining layer is about 5 md, a value that is one order of magnitude less than the permeability of the aquifer of 50 mdarcy estimated from a slug test.

Although phase lag increases slightly between 1 and 2 cycles/day, nothing else suggests that any attenuation occurs due to limited groundwater flow between the aquifer and the borehole. Assuming that dimensionless frequency W is less than 0.1, the lower bound on permeability for the aquifer is 60 mdarcy, a value slightly greater than the permeability of 50 mdarcy inferred from the slug test data.

Well SC2

The response of SC2 to air pressure shown in Figure 10 indicates that both the barometric efficiency and phase are relatively flat over the observed frequency band. Because the response lacks any strong trend, interpretation of the response is somewhat ambiguous. The figure shows two interpretations of the response. In the first interpretation (model 1), the static-confined response is observed over the entire frequency band. The barometric efficiency is a flat 0.78, Q is greater than 41ω, and R is not indentifiable. Alternatively, water table effects begin to slightly influence water well response at the low end of the observable frequency (model 2). In this interpretation, the static-confined barometric efficiency is 0.74 and the values for Q and R are 10ω and less than 1.0ω, respectively.

Table 2 shows the air and hydraulic diffusivities inferred from Model 2. The lower bound on pneumatic diffusivity is nearly the same as the pneumatic diffusivity estimated at TF; the hydraulic diffusivity of the partial confining layer is considerably lower. The specific storage of the aquifer is estimated to be (RA, 1988) 4.9×10^{-6} m^{-1}. If I assume that the specific storage of the confining layer and the aquifer are the same, the vertical permeability of the confining layer is estimated to be 5×10^{-2} mdarcy, indicating that the confining layer is composed of considerably different material than the aquifer. This inference is consistent with the lithology at the site: the well taps a fractured basalt overlain by glacial till [*Farrar et al.*, 1985].

Once again, there is no observable attenuation of response due to limited hydraulic communication between the quifer and the borehole. The lack of observable attenuation indicates that aquifer permeability is greater than 90 md; the slug test data suggest that aquifer permeability is 2×10^{7} mdarcy, a value much larger than this lower bound.

CONCLUSIONS

The response of water levels in wells that tap partially confined aquifers to atmospheric loading is dependent on the elastic and fluid flow properties of the aquifer as well as the material overlying the aquifer. Owing to the hydraulic properties of the aquifer and confining layer and the pneumatic properties of the unsaturated zone, water well response cannot be expected to be independent of frequency. Attenuation and amplification of the static-confined response to atmospheric loading can occur in theory and is observed in the wells examined here. Phase lags and advances observed in response to atmospheric loading also have a theoretical basis.

In many instances, the response of a well can be divided into three frequency bands. The response at low frequencies is independent of aquifer permeability and depends on the confining layer and unsaturated zone diffusivities. Attenuation and amplification as well as phase lags and phase advances are possible in this frequency band. The response at intermediate frequencies is dependent on the elastic properties of the aquifer and is independent of fluid flow properties; it is characterized by a flat barometric efficiency and phase. The response at high frequencies is independent of confining layer and unsaturated zone diffusivity and is strongly dependent on aquifer permeability. It is characterized by increasing attenuation and phase lag with increasing frequency. The width of separation between the high- and low-frequency response (i.e., the width of the intermediate-frequency band) is dependent on the well radius, the aquifer transmissivity, and the confining layer thickness and hydraulic diffusivity.

The theoretical response can be used in conjunction with the observed response of water wells as a function of frequency to yield estimates or place bounds on the fluid flow parameters within the aquifer, confining layer and unsaturated zone. For the wells examined, water well response to atmospheric loading does not yield much information on aquifer permeability; it is possible only to obtain a lower bound for this flow parameter. In low-permeability environments, however, the response of water wells to atmospheric loading may prove useful in estimating aquifer permeability.

Water well response, for the wells examined here, does serve to yield useful estimates of confining layer hydraulic diffusivity and the pneumatic diffusivity of the unsaturated zone. If the site lithology indicates that the specific storage of the confining layer is close to the value of specific storage of the aquifer, it is also possible to make an estimate of the confining layers vertical permeability. Estimates of these parameters are usually difficult to obtain using a conventional techniques and are valuable for purposes of water resource assessment and studies of contaminant migration in the near surface.

APPENDIX: SOLUTION TO THE DRAWDOWN IN A WELL WITH PERIODIC DISCHARGE TAPPING A PARTIALLY CONFINED AQUIFER

The drawdown within an aquifer that is partially confined in response to periodic discharge from a well is assumed to be governed by the following equation and boundary conditions:

$$\frac{\partial^2 s}{\partial r^2} + \frac{1 \partial s}{r \partial r} - \frac{K' s}{K b b'} = \frac{S_s \partial s}{K \partial t} \qquad (A1a)$$

$$s(\infty, t) = 0 \qquad (A1b)$$

$$\lim_{r \to 0} \frac{r \partial s}{\partial r} = \frac{\omega r_w^2 x_0}{2Kb} \sin \omega t \qquad (A1c)$$

No initial condition is imposed because I seek the periodic steady state solution. This problem is readily solved employing complex notation. Taking s to be complex

$$s(r, t) = F(r) \exp(i\omega t) \qquad (A2)$$

and substituting in (A1) I obtain

$$F'' + \frac{F'}{r} - \left\{\frac{K'}{Kbb'} + \frac{S_s i\omega}{K}\right\} F = 0 \qquad (A3a)$$

$$F(\infty) = 0 \qquad (A3b)$$

$$\lim_{r \to 0} \frac{r \partial F}{\partial r} = \frac{-i\omega r_w^2 x_0}{2Kb} \qquad (A3c)$$

where the prime implies differentiation and all exponential terms have been divided out. Equation (A3) is an ordinary differential equation with radial symmetry. Its general solution is given by [*Tranter*, 1968]

$$F = C_1 I_0(r) + C_2 K_0(r) \qquad (A4)$$

where C_1 and C_2 are constants determined by the boundary conditions and I_0 and K_0 are modified Bessel functions of the first and second kind of order zero, respectively. The boundary condition (A3*b*) requires that C_1 equals zero. The solution for drawdown at the radius r_w is

$$F_w = i0.5 W x_0 K_0\{[W^2(S^2 + 1/q^2)]^{0.25} \exp[i0.5\{\tan^{-1}(qS)\}]\} \qquad (A5)$$

The complete solution is given in (13).

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