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The Value of Grain-size Hydraulic Conductivity Estimates: Comparison with High Resolution In-situ Field Hydraulic Conductivity

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Abstract. Hydraulic conductivity is commonly estimated from grain size characteristics of unconsolidated sediments. We present an extensive set of air-permeability and grain size measurements from a heterogeneous aquifer. These data are unique in that the hydraulic conductivity measurements are made on undisturbed in-situ sediment and, like the grain size measurements, have a small sample volume size (<1200 cm^3). Hydraulic conductivity estimated from grain-size has a higher mean and a lower variance than direct measurements. While grain size estimates of hydraulic conductivity do not correspond well to field measured hydraulic conductivity, spatial correlation of the two data sets is quite similar.

1. Introduction

Hydraulic conductivity is often the greatest source of uncertainty in predictive transport models of solute transport in groundwater. Pumping tests and slug tests performed in wells are the common methods of measuring hydraulic conductivity in aquifers. But because well tests can be prohibitively expensive, impractical, and sample relatively large volumes, hydraulic conductivity is frequently estimated from grain-size characteristics.

Equations relating hydraulic conductivity to grain-size have a long history beginning with Hazen (1892), who developed the well known empirical equation :

$$K= C(d10)2 \tag{1}$$

where: K is hydraulic conductivity at 20°C (cm/s), d10 (mm) is the tenth percentile grain size by weight, and C is a dimensionless coefficient affected by a variety of factors. Expressions relating hydraulic conductivity to grain-size also employ parameters such as mean grain size, sorting, and porosity (Fair and Hatch, 1933; Krumbein and Monk, 1942; Harleman et al., 1963). Although the Hazen equation was developed for sand-sized sediment and not for sediment with gravel or clay, such as that examined in this study, we apply the Hazen method in this study because it is widely accepted and used. We do not apply other methods because we did not measure porosity, grain shape, or packing factors and the grain size probability distributions do not show Gaussian distributions.

Field and laboratory studies of hydraulic conductivity - grain-size correlation have been widely published for many decades and demonstrate that, while accurate empirical expressions can sometimes be established for a specific source material, no expression has been found to accurately estimate hydraulic conductivity across the range of naturally occurring sediment types.

The reasons for the poor predictive performance of equations such as (1) are not fully understood. Numerous authors have argued that hydraulic conductivity should increase with effective grain size and decrease with grain size variability. Empirical studies have generally found this to be true (Shepherd, 1989). But hydraulic conductivity is sensitive to other parameters such as sediment stratification (Pryor, 1973), low weight percentage fines (Alyamani and Sen, 1993), and cementation (Uma and Loehnert, 1994), commonly left out of hydraulic conductivity estimation equations. A further difficulty in predicting hydraulic conductivity from grain size is the occurrence of different materials having similar grain-size distributions but different hydraulic conductivity values (Cheng et al., 1999).

Most comparisons of grain-size and field hydraulic conductivity correlation rely on laboratory hydraulic conductivity measurements of repacked or cored sediment and do not measure in-situ hydraulic conductivity values (Wolf et al., 1991). Sediment properties affecting hydraulic conductivity, such as grain cohesion and layering, can be disturbed during laboratory measurement. So laboratory measurements may not yield conductivity measurements reflecting actual field hydraulic conductivity. Because hydraulic conductivity is extremely variable and sensitive to small changes in pore structure, one can expect conductivity values to most accurately reflect in-situ conditions when in-situ measurements are taken (Shan, 1995). Some studies have compared grain size characteristics to hydraulic conductivity measured in-situ by either slug tests or pumping tests. These studies avoid the problems of sampling undisturbed sediment but introduce scaling problems because the support volume (measurement volume) of slug tests and pumping tests is typically four to five orders of magnitude larger than the size of sediment samples. As recent studies have shown, support volume of a measurement can strongly affect measured hydraulic conductivity values (Tidwell et al., 1999).

This study examines hydraulic conductivity and grain-size correlation using data with small support volumes, <1200 cm^3. The hydraulic conductivity measurements are taken in-situ, which allows us to examine whether grain-size data can yield accurate data on fine scale (<1 meter) hydraulic conductivity variations. Hydraulic conductivity values derived from grain-size have a very different origin than flow-measurement based hydraulic conductivity values and this paper examines the commonly observed disparity in their statistics.

2. Study Site

The study aquifer is a shallow (<15 m depth) braided stream Pleistocene deposit. All data reported in this study are from a pit 1 km southeast of the Columbus Air Force Base (CAFB) test site, where detailed hydrogeologic investigations were performed in the same formation (Boggs et al., 1992). The CAFB study found conductivity to be spatially heterogeneous with ln (hydraulic conductivity) variance = 4.4.

Aquifer sediments are semi-consolidated sands and gravels with fine horizontal layering of texture and color. At the quarry, the upper aquifer portion was exposed along vertical

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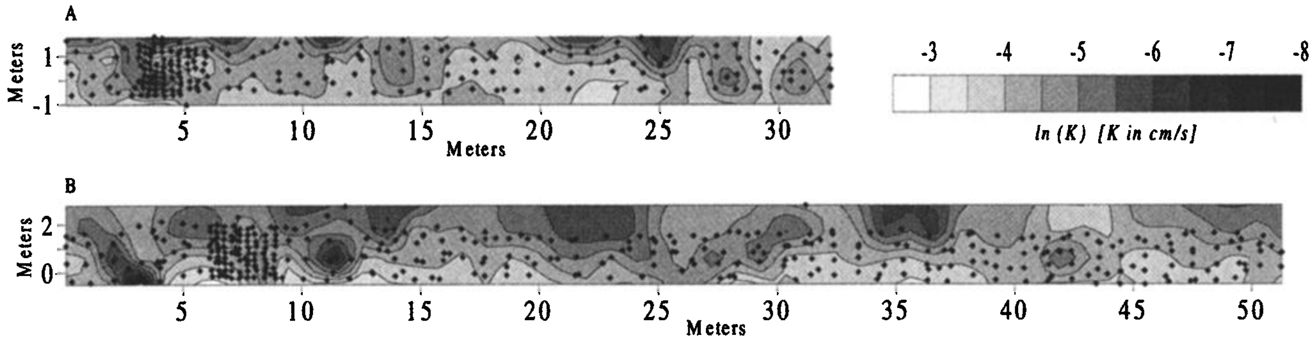

Figure 1. Sample locations and ln (hydraulic conductivity). Darker shading indicates lower ln(hydraulic conductivity), ● = sample location, (A) North-south oriented quarry wall (B) East-west oriented quarry wall.

walls 3-4 meters high. Hydraulic conductivity tests and grain-size analyses were made on a 50 meter wall oriented east-west and a 30 meter wall oriented north-south. Sample locations were selected by a combination of regular sampling on coarse and fine grids and random sampling (Figure 1). The X, Y, Z coordinates of each measurement location were surveyed and, because the walls were not exactly planar, sample locations were projected onto a plane for spatial analysis. Projection distances were less than 0.5 meters.

Aquifer sediments exposed at the quarry were massive with little facies delineation. The upper 1 meter of the aquifer has higher clay content, weathering, and veins of precipitated calcite. Exposed sediment was a gravel-sand mixture with a small fines proportion (<2%) that, while spatially variable in texture, showed relatively few facies delineations. Lenticular bodies of non-cohesive, well-sorted, medium sands made up about 5% of the total exposure.

3. Methods

Sediment samples were collected from quarry walls at positions shown in Figure 1. Approximately 1200 cm^3 of sediment was collected at each location and samples were sealed to preserve moisture. In the laboratory, sediment moisture content and grain size distribution were measured. d_{10} grain-size and Hazen's equation were used to estimate K.

An air permeameter using compressed air as a fluid source was used to measure hydraulic conductivity. Air flows from a rubber tip into the sediment and then back out to the free surface. When pressure and flow rate stabilize, usually within 10 seconds, flow rate, air pressure, and air temperature are recorded. Similar instruments have been used in previous hydrogeologic and petroleum reservoir studies (Eijpe and Weber, 1971; Sharp et al., 1994).

The air permeameter measured permeability in the range of 300 to 150,000 md (a ln(K) range of –8.1 to –1.9, K in cm/s). Above this range, flow rates were too high, leading to non-linear flow and below this range, air flow was too small to be measured by the permeameter.

The support volume of the air permeameter is controlled primarily by tip seal geometry (Goggin et al., 1988). Our permeameter tip had an internal radius of 0.15 cm and an external radius of 1.65 cm which gives a sample volume estimate of approximately 15 cm^3.

To prepare for hydraulic conductivity measurement, loose material was brushed from the surface of the walls. A 3-4 cm diameter area was selected on which to apply the permeameter tip. At some locations, sediment was too cohesionless to permit a tight seal and these locations were abandoned, approximately 10% of surveyed locations At locations where large gravel entirely blocked the permeameter tip, alternate locations were chosen within 10 cm. .

Soil moisture affects air flow in unsaturated materials because water fills pores, reducing pathways available for air flow. Davis et al.(1994) found at low moisture content, less than 5%, permeability measurement error is also less than 5%. Springer et al. (1998) found permeability of a silty sand decreased by just 4% when moisture content increased from 1% to 12.5%. We therefore excluded from our analysis sample locations having moisture content >6% by weight, which excluded about 5% of the samples. A total of 588 sample locations met all test criteria and yielded permeability, grain size distribution, and location measurements.

Using Equation 2, Goggin et al. (1988) and Davis et al. (1994), measurements were converted to permeability,

$$k = \frac{qP_l\mu}{aG\frac{P_l^2 - P_0^2}{2}} \qquad (2)$$

where:

a = internal radius of injection tip (L)
G = geometric flow factor = 4.1 [from Fig. 5 Goggin et al. (1988) b=16.5 mm, a = 1.5 mm]
k = permeability (L^2)
μ = dynamic viscosity of air ($ML^{-1}T^{-1}$)
P_0 = atmospheric pressure ($ML^{-1}T^{-2}$)
P_l = injected air pressure ($ML^{-1}T^{-2}$)
q = air flow rate (L/T)

Flow rates are converted to std. temperature and pressure by ideal gas laws. Permeability is converted to hydraulic conductivity by accounting for viscosity and density of water at 20°C (Equation 3)

Table 1. ln (K) Measurement Statistics

Statistic	Hazen	Air Permeameter
N	588	588
Mean	-2.35	-3.96
Variance	0.30	0.76
Horizontal Corr. Length (m)	1.5	3.0
Approximate Sample Volume (cm^3)	~1200	~15

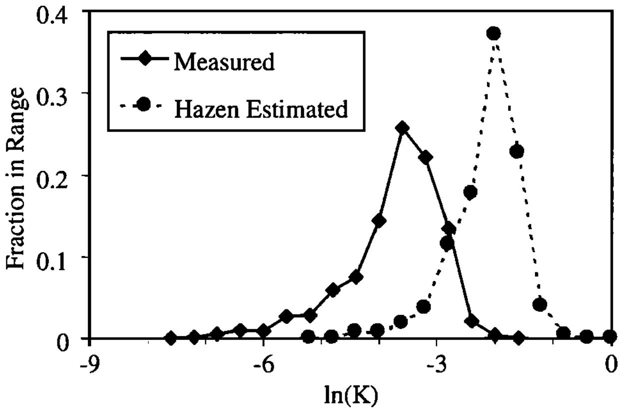

Figure 2. Histogram of measured and Hazen ln(K).

$$K = \frac{kg\rho}{\mu_w} \tag{3}$$

where:

g = gravity (LT^{-2})

k = permeability (L^2)

K = hydraulic conductivity (LT^{-1})

μ_w = dynamic viscosity of water ($ML^{-1}T^{-1}$)

ρ = bulk density of water (ML^{-3})

To test the air permeameter's accuracy and reliability we measured hydraulic conductivity of nine porous, permeable, cinder blocks. The cinder blocks were solid with sides from 15-45 cm in length. The variety of block materials and types gave a permeability range spanning the lower 65% of the permeability range found at the field site. Cores of each block were taken and sent to an outside lab for permeability tests. In a whole population comparison of the ln(k) sample populations (k in md), a t-test shows the hypothesis that our laboratory mean log permeability (8.02) is the same as the outside lab's mean (7.94) is accepted at the 95% confidence level. Of the nine test blocks, t-tests indicate the hypothesis that the air permeameter mean ln(k) is the same as the outside lab mean ln(k) can be rejected for 3 of the 9 test blocks (3, 4, and 9). But measured air permeability shows no consistent bias to high or low permeability values. There are several possible sources for the differences between our permeability values and the outside lab values including sample error and analysis technique. Because the hypothesis tests generally show good agreement between the two measures, we assume that air permeameter measurements provide a direct and reasonably accurate measure of true permeability.

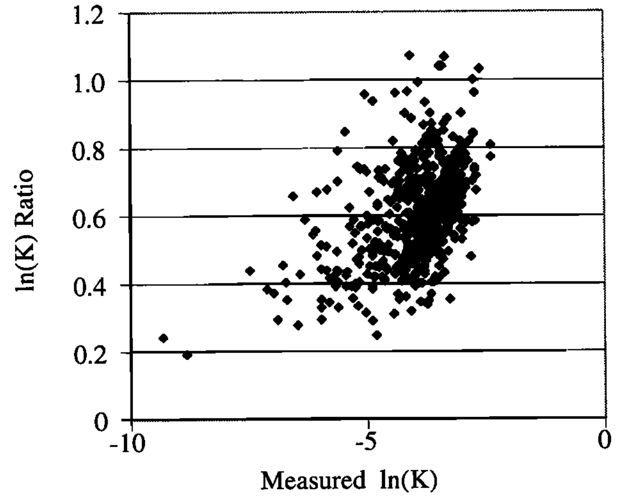

Figure 3. Ratio of air and Hazen ln(K) (N=588). Values <1.0 indicate overestimation of ln(K) by Hazen's equation.

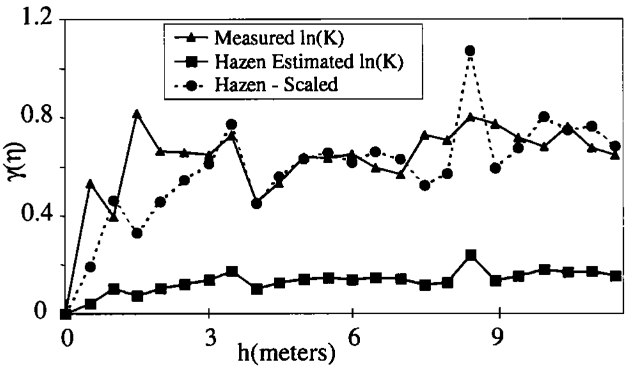

Figure 4. Sample horizontal variograms for measured ln(K) and for Hazen estimated ln(K) on the north-south wall. Dotted line is the Hazen variogram scaled to have the same variance as the measured ln(K) data..

4. Results

Aquifer sediment has an average grain-size of 1.1 mm, average d_{10} of 0.3 mm, and is over 40% gravel (>2 mm). Summary statistics for the measured and Hazen estimated ln (hydraulic conductivity) values are given in Table 1. The support volume of the Hazen ln (K) values is bigger than the support volume of the air permeameter measurements. As compared to the air permeameter values, the Hazen ln (hydraulic conductivity) values have a higher mean and lower variance. Histograms are shown in Figure 2. The difference in mean ln(K) translates to the Hazen equation overestimating hydraulic conductivity by a factor of 4.4. As seen in Figure 3, the Hazen equation's overestimation of hydraulic conductivity is greater for lower values.

Sample variograms were calculated for the measured and Hazen estimated ln (K) values, with Figure 4 showing variograms for the north-south wall. The east-west wall showed similar horizontal variograms. Except for having a smaller variance, the Hazen estimated ln (K) variogram matches well with the measured ln (K) variogram. This similar structure can be see in the dotted line of Figure 4, which is the Hazen variogram scaled to have the same variance as the measured hydraulic conductivity values. Both the Hazen estimated and measured ln (K).variograms show significant spatial variability at small horizontal separation distances (<3 meters for Hazen and <1.5 meters for air permeameter values).

5. Discussion

There are both limitations and benefits to using air permeameter measurements comparatively. Limits to the air permeameter sampling range prevent accurate measurement of hydraulic conductivity greater than 0.3 (cm/s) or lower than 0.0001 (cm/s). Because the air permeameter's upper limit is relatively high, we expect that more hydraulic conductivity values are missed below the lower limit than are missed above the upper limit. A benefit to using an air permeameter is that, unlike most studies, the grain size K estimates are compared to hydraulic conductivity measurements having a support scale one or two orders of magnitude smaller.

Taking the air permeameter measurements as indicative of true in-situ hydraulic conductivity values, two conclusions can be drawn about the Hazen equation's ability to estimate hydraulic conductivity. First, the Hazen equation yields hydraulic conductivity values that are much too high. Although the Hazen coefficients (C and the exponent in Eq. 2) could be empirically fit to improve estimation, we do not do that here because the our goal is to assess the utility of Hazen values as stand-alone data. Second, the Hazen values have less variability than they should, particularly if lower hydraulic conductivity values are not sampled. Whether the statistical differences between Hazen and measured ln(K) are affected by the larger Hazen support volume is unknown. Although mean hydraulic conductivity typically increases and hydraulic conductivity variance typically decrease with larger support volumes, the opposite can also occur (Zlotnick et al., 2000). Because some visible sediment layers were thinner (1-5 cm) than the support scale of the Hazen values (~11 cm) the Hazen values may have averaged out some variability, resulting in a lower variance relative to the air permeameter measurements.

The Hazen equation's failure to reproduce low hydraulic conductivity values indicates that d_{10} grain size is less important for determining hydraulic conductivity of tight sediments than other factors not included in the Hazen equation. Two factors possibly controlling hydraulic conductivity of tighter sediments are a small silt/clay fraction (<2%), which was present in almost all of the sediment samples, and iron oxide cementation, which was visible at some locations. All factors affecting hydraulic conductivity are incorporated into air permeameter measurements because they are based on flow through in-situ sediment.

Spatial correlation of Hazen estimated ln (hydraulic conductivity) matches the measured spatial correlation quite well in both horizontal directions. This finding has practical importance because it indicates the potential for using grain size analyses of sediment samples to determine spatial correlation functions for hydraulic conductivity. Although the Hazen equation does not have good predictive performance for estimating hydraulic conductivity, the accurate estimation of hydraulic conductivity correlation lengths may justify sediment collection and grain size analysis for some groundwater projects.

Finally, these field results show that significant spatial variation of hydraulic conductivity exists at scales from 0.1 to 5 meters in the Columbus aquifer. In aquifers like this where strong fine scale variability exists, many (perhaps hundreds or thousands) of hydraulic conductivity measurements may be needed to adequately characterize the aquifer for mass transport modeling and prediction. The results of this study indicate that attempting to use grain size as a surrogate for direct measurement of K may yield accurate values for the spatial correlation lengths of hydraulic conductivity but is not likely to yield accurate estimates of hydraulic conductivity itself. Mass transport prediction will require extensive direct measurements of fine scale (<1 meter) hydraulic conductivity at complex sites like the one examined in this paper.

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