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Inferring spatial correlation of hydraulic conductivity from sediment cores and outcrops

Jack Eggleston and Stuart Rojstaczer

Center for Hydrologic Science, Division of Earth and Ocean Sciences, Duke University, Durham, NC

Abstract. Spatial variation of hydraulic conductivity is often assumed to follow patterns of sediment texture. This study examines whether sediment texture observed in cores and outcrops can be used to infer spatial correlation of hydraulic conductivity. The measures of spatial correlation that we consider, correlation length and Hurst coefficient, are commonly used for interpolating conductivity at unmeasured locations and making stochastic estimates of dispersivity. Results indicate that, when comparing hydraulic conductivity values derived from sediment texture to those measured by pumping tests, horizontal correlation lengths are approximately the same but vertical correlation does not correspond well. The results also suggest that sediment information may be useful for setting spatial limits to scaling behavior that is assumed with fractal interpretation of hydraulic conductivity variability.

Introduction

Understanding groundwater flow and contaminant transport in aquifers requires characterization of subsurface heterogeneity, particularly the spatial variation of hydraulic conductivity. The large effort and expense often required to characterize groundwater systems usually forces one to make do with sparse observational data and rely heavily on interpolation to construct maps of subsurface variability. The approach of sparse data collection and subsequent interpolation is a questionable one because conductivity can vary over several orders of magnitude, even within a single aquifer. As a result, one's ability to predict the fate of contaminants in groundwater and to implement efficient and effective remediation strategies is severely limited.

If the standard approach of sparse direct measurements of hydraulic conductivity (K) is an ineffective means of characterization, are there alternate means of indirectly inferring spatial variability in conductivity? Over the past two decades a large body of research has been devoted to using geologic information to indirectly infer spatial variability in conductivity. A common approach is to determine the depositional structure of a groundwater system and assign K values assuming that they are controlled by lithology. But questions still exist about how predictions of conductivity can be improved by incorporating information on sedimentary structure. Oddly, there have been few efforts at direct field comparison between spatial variations in sediment texture and hydraulic conductivity. The exceptions are a handful of studies relating air permeability to depositional patterns in outcrops [Davis et al., 1993; Hurst and Rosvoll, 1991].

In this study we compare spatial variations in hydraulic conductivity and in sediment texture. We test whether two measures of aquifer heterogeneity, spatial correlation length and Hurst coefficient, can be inferred from shallow aquifer exposures and sediment cores. This comparison gives us a simple means to examine the worth of using sediment core and outcrop information in characterizing spatial correlation of conductivity.

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Site Description

The study aquifer is a shallow (<15 m in depth) braided stream deposit site underlying the Columbus Air Force Base in Mississippi. We chose the site because previous detailed hydrogeologic investigations have been performed there [Rehfeldt et al., 1989, 1992; Boggs et al., 1990] and nearby quarries allow sediment sampling and mapping. At a gravel quarry located 1000 m SE of the well field, we collected additional sediment samples and mapped sediment structure.

At the well field, 2451 flowmeter measurements describe the spatial distribution of horizontal hydraulic conductivity (K = cm/s) [Boggs et al., 1990; Rehfeldt et al., 1992]. Six sediment cores were also taken within the well network (Figure 1). The 2451 measurements have mean ln (K) = -5.4. ln (K) variance = 4.4, which is high compared other sites having extensive conductivity measurements in an alluvial aquifer (Borden: 0.38 [Sudicky, 1986], and Cape Cod: 0.24 [Hess et al., 1992]).

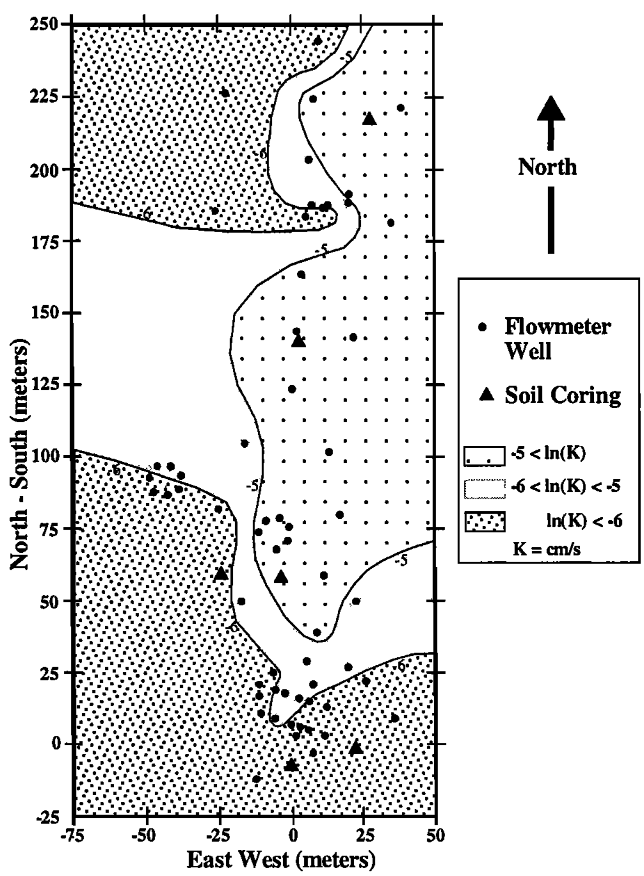

Figure 1. Well and core locations at the well field. Lighter shading indicates regions of greater hydraulic conductivity as determined by vertically averaged flowmeter well measurements.

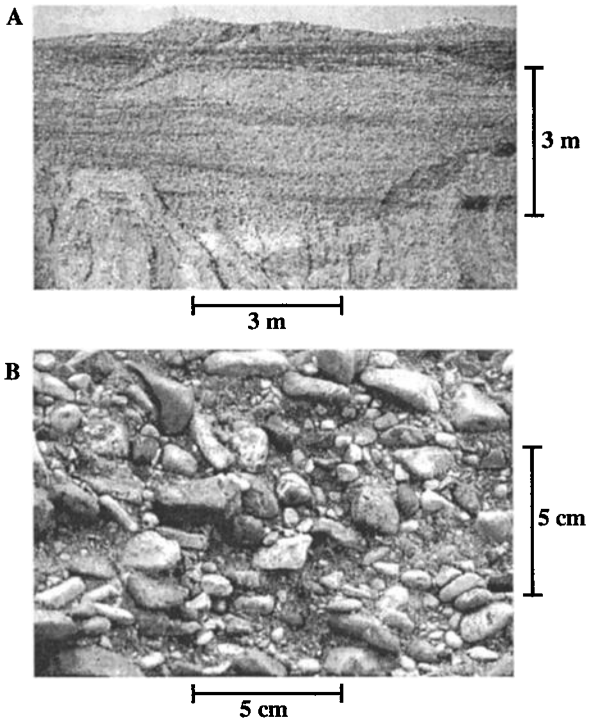

Figure 2. Quarry sediment. A) Upper 3 m of aquifer formation. B) Rounded gravels of 1-4 cm diameter are throughout the aquifer formation and account for 40% of the material by weight.

At the quarry 1 km SE of the well field the upper 3 meters of the aquifer was exposed and we collected 42 sediment samples. The aquifer sediments are semi-consolidated sands and gravels with fine horizontal layering of texture and color (Figure 2).

An earlier study, at a different quarry 1.5 NE of the wellfield, mapped sediment facies on a 3 x 20 m exposure and estimated correlation scales by assigning K values based on sediment appearance and constructing a variogram [Rehfeldt et al., 1992]. These correlation lengths were smaller than those obtained from the flowmeter K data by a factor of ten and it was suggested that future studies should collect samples for grain size analysis.

Methods

Flowmeter Measurements of Hydraulic Conductivity

At the well field, measurements of hydraulic conductivity were made by a variety of methods, but only the flowmeter measurements are considered here because they are the most reliable and provide the greatest spatial detail [Boggs et al., 1990]. The length scale of a flowmeter measurement can be calculated as follows:

$$r = 1.5\sqrt{\frac{K_p H t}{S}} \tag{1}$$

where: r = radius of pumping influence, K_p = depth-averaged hydraulic conductivity, H = aquifer thickness, S = storage coefficient, and t = time to steady state water levels. Based on a range of parameter values from wells located in high and low conductivity regions [Rehfeldt et al., 1989, table 5-1] r is calculated to range from 4 to 50 meters.

Estimating Hydraulic Conductivity from Grain Size

We used all 214 sediment samples collected by Boggs et al. [1990] around the well field for calculating a horizontal variogram. For all other analyses of well field core we used a subset of samples (n=74) taken within the flowmeter well area. From grain size distributions determined by sieving we estimated K for each sediment sample. For samples with a narrow particle distribution, Hazen's equation (Eq. 2) was used [Fetter, 1988].

$$K = cd_{10}^2 \qquad \text{if } U < 5 \tag{2}$$

where: K is hydraulic conductivity (cm/s), c = 100 (cm^{-1}s^{-1}), $U = d_{60}/d_{10}$, and d_{10} & d_{60} are the 10% and 60% grain diameters (cm). For samples with a wide grain size distribution, Seiler's equation was used following Boggs et al. [1990].

$$\begin{aligned} K &= C(U)d_{10}^2 \qquad \text{if } 5 < U < 17 \\ K &= C(U)d_{25}^2 \qquad \text{if } U > 17 \end{aligned} \tag{3}$$

where $C(U)$ is a constant from Table 2-10 in Boggs et al. [1990].

Equations predicting K from grain size generally give poor estimates, with errors often more than an order of magnitude [Shepherd, 1989]. However, it may still be possible to use K estimates from Equations 2 and 3 to infer spatial correlation of the true K field if estimation errors are uncorrelated in space. Although it is impossible to know a-priori if estimation errors will exhibit spatial correlation, they will show spatial correlation if sediment patterns have large scale trends and the magnitude of error is dependent on d_{10}.

Table 1. ln (K) Statistics

Data Source	#	Mean	Variance	Correlation Length (m) Vertical	Horizontal
Well Field					
Flowmeter	2451	-5.4	4.4	4.1	57
Grain Size	74	-3.0	3.5	<0.4	68
Quarry					
Grain Size	42	-2.4	1.5	<0.6	12

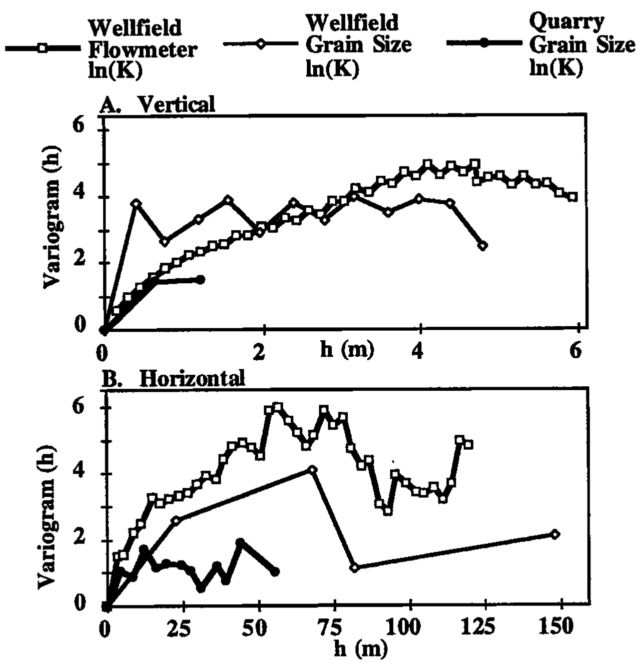

Figure 3. Experimental ln (K) variograms. Squares are for flowmeter ln (K) from the well field. Diamonds are for grain size derived ln (K) at the well field. Circles are for grain size derived ln (K) at the quarry. (A) Vertical, same well and same core pairs only. (B) Horizontal, horizontal isotropy is assumed.

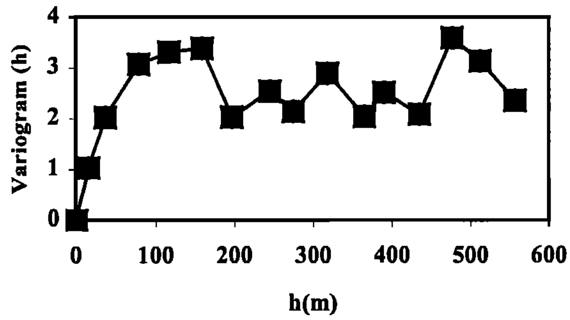

Figure 4. Long-range horizontal variogram for grain size derived ln (K). Data include the quarry sediment samples and 214 sediment samples from a wide area around the well field.

Analysis of Hydraulic Conductivity Spatial Correlation

Experimental variograms are one of the statistical measures used in this study to express spatial correlation of K. The variogram, $\gamma(h)$, is given by Equation 4.

$$\gamma(h)=\frac{1}{2n(h)}\sum_{i=1}^{n(h)}\left[z(x_i+h)-z(x_i)\right] \tag{4}$$

where h is the lag, $n(h)$ is the number of pairs, and z is measured ln (K) at locations (x_i) and (x_i+h). We calculate vertical and horizontal variograms (first calculated by Boggs et al. [1990]) and visually determine correlation lengths.

In cases where spatial variability continues to increases with the sampling scale, the variogram does not reach a sill and does not accurately express long range correlation. We then used an alternative measure of spatial variability to better express long range correlation; 'rescaled range' analysis. Rescaled range analysis assumes K variation is a scaling phenomenon and that correlation length and variance increase indefinitely with spatial scale [Turcotte, 1997].

If a ln (R/S) plot is linear, then the slope of the plot is the Hurst coefficient (H). For data with a Gaussian distribution the Hurst coefficient varies from 0 to 1. H values < 0.5 indicate negative correlation, H values > 0.5 indicate positive correlation, and H = 0.5 indicates random noise. We perform R/S analysis in the vertical direction and results for wells and sediment cores are averaged for each group.

Fractal Analysis of Sediment Layering

The quarry wall exhibits horizontal layers of sand and gravel conglomerate (Figure 2A). The layers are from 1 cm to 2 m thick and persist laterally for up to 10 m. Close examination of

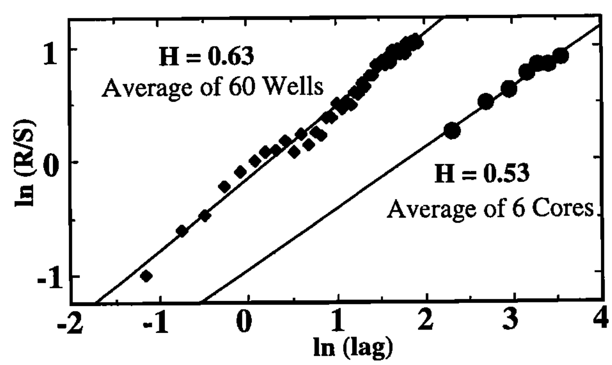

Figure 5. Vertical rescaled range (R/S) analysis. Fitted slope is Hurst coefficient (H). (a) Diamonds are average R/S values for flowmeter K from 60 wells. (b) Circles are average R/S values for grain size K from 6 cores in the well field.

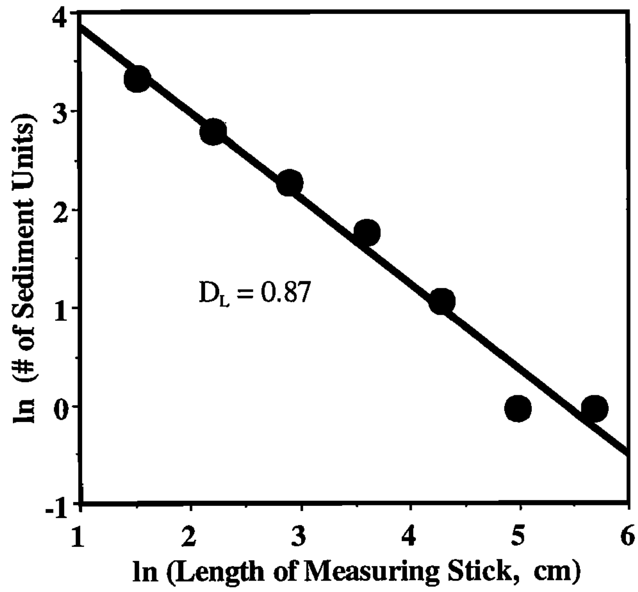

Figure 6. Fractal analysis of sediment layers seen in Figure 3a. Y axis is the natural log of the number of sediment layers with thickness ≥ corresponding X axis length.

the sediment layers reveals internal layering defined by contrasting sediment texture, color, and cementation. To quantify this increase of layering detail with the scale of observation, we took a fractal approach. On a vertical slice from the center of Figure 2A, we counted the number of sediment units ($N(l)$) with a thickness greater than length (l). A sediment unit is contained between two bounding surfaces. Bounding surfaces were identified based on contrasts in sediment texture and color, with higher contrast surfaces bounding thicker sediment units. A plot of ln ($N(l)$) vs. ln (l) was then constructed. The negative slope of this plot is a fractal dimension (D_L) describing the degree of imbedded vertical layering.

$$D_L=\frac{\ln\left(r+\frac{1}{N}\right)}{\ln(r)} \tag{5}$$

A similar approach was used by Rothman et al. [1994] to describe turbidite layers. If there are no imbedded sediment units, as in the ideal cases of a single homogeneous unit or if all units have equal thickness, then $D_L = 0$. For the case in which a sediment package is divisible into r subunits and each of those sediment unit is in turn divisible into exactly r sub-units, D_L approaches 1.0. If imbedded layers are present, but do not completely occupy each sediment unit, then $0 < D_L < 1$.

To infer K properties from sediment layering requires an assumption that K variability is controlled by changes in lithology or sediment type. If this is true, then the spatial scales at which lithology changes are also the scales at which K changes. Knowing the spatial scale of K variability is important for a number of reasons including establishing limits to the use of scaling relations for interpolating K.

Results

Similar to the well field, the quarry has a 2-3 meter layer of silt/clay overlying a sand and gravel unit ~12 meters thick. Surface elevations at the two sites differs by 0.6 meters and the average grain size distributions of the two sites are quite close (with gravels making up 40% of the aquifer at both sites (Figure 2B) and mean d_{10} grain sizes of 0.27 mm and 0.31 mm respectively). Considering the identical lithologic transitions and similar grain size distributions, we expect that aquifer materials at the quarry and the well field are part of the same deposit.

Hydraulic conductivity values calculated from grain size are higher and less variable than conductivity values measured by flowmeter. Table 1 gives ln (K) statistics. Grain size K shows greater variability at the well field than at the quarry although mean K from the two locations is similar. The grain size derived ln (K) values are higher than the flowmeter ln (K) measurements.

Horizontal variograms indicate that grain size K is correlated over the same lateral distance as flowmeter K (Figure 3). Vertical correlation, however is not the same for grain size and flowmeter K. Vertical variograms show a vertical correlation length of about 4 m for flowmeter K but almost no spatial correlation in grain size K. The long-range horizontal variogram in Figure 4 suggests that K variability does not increase at distances greater than 200 meters. More data with greater spatial coverage is needed to confirm this initial finding.

Both flowmeter and grain size K have log-linear slopes in the R/S plot (Figure 5). The R/S analysis shows more long-range vertical correlation for the flowmeter K than for the grain size K. H coefficients for the 60 flowmeter wells and 6 sediment cores are 0.63 and 0.53, respectively. The value of 0.63 is higher than that found by Liu et al. [1996] who report an average H value of 0.54 for a subset of 12 of the Columbus wells.

Most individual wells and cores have ln (R/S) plots that are not linear and have much more scatter than the averaged plot of Figure 5. H coefficients for individual flowmeter wells, from linear regression of the ln (R/S) plots, range from –0.1 to 2.8 and have a standard deviation of 0.48. This variability adds ambiguity to any fractal interpretation of K at the site.

At the quarry, visual analysis of the number of sediment units as a function of length over 1.5 orders of magnitude yields a plot that is log-linear down to a scale of 4 cm with $D_L = 0.87$ (Figure 6). This implies that K also has significant variability, possibly fractal in nature, down to a vertical scale of 4 cm (below the minimum 15 cm scale of the flowmeter measurements).

Discussion and Conclusions

Although the sampling volume is larger for the flowmeter measurements than for the sediment samples, the ln (K) variance is unexpectedly higher for the flowmeter values. The mean ln (K) from grain size estimates also shows a positive bias relative to the flowmeter mean ln (K). This positive bias and the presence of large scale trends at the Columbus site raises a suspicion that grain size K estimation errors are spatially correlated, which could cause spatial correlation of grain size K and flowmeter K to differ. However, our estimated horizontal correlation lengths of flowmeter K and grain size K agree to within 20%.

Vertical correlation is not the same. Flowmeter measurements of K have much more vertical persistence than the grain size derived K, with a correlation length of 4 m for flowmeter K and essentially no vertical correlation for grain size K. This difference in vertical correlation may be caused by non-uniform flow during flowmeter testing. If vertical flow is induced by pumping, then K values are vertically averaged over an interval larger than the assumed 15 cm and so should show correlation over a greater vertical distance.

Fractal analyses of vertical correlation also show greater correlation in flowmeter measurements than in grain size derived K. Rescaled range analysis of well field data shows positive correlation (H=0.63) in flowmeter K and almost no correlation (H=0.53) in grain size derived K. This may be due to the larger sample volume size of the flowmeter measurements. The fractal analysis of sediment layering suggests that K is variable down to a scale of at least 4 cm, well below the minimum 15 cm vertical scale of the flowmeter measurements.

The absence of long-range correlation in the horizontal variogram indicates that no further increase in K variance or correlation scale should be expected at distances beyond 200 meters. This is in contrast to the type of scaling discussed by Neuman [1994] that suggests that K variation increases indefinitely with scale. In the absence of further information indicating that K variability increases beyond a horizontal scale of 200 meters at the site, scaling laws should not be used to obtain a K variance for the regional domain that is larger than that observed within the well field.

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J. Eggleston and S. Rojstaczer, Center for Hydrologic Science, Duke University, Durham, NC 27708-0230. (e-mail: jre4@acpub.duke.edu; stuart@geo.duke.edu)

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